16.2

3.

(a) $F(r(t)) = F(t, t^{-1}) = (t^{-2}, t^2)$

$r'(t)dt = (1, -t^{-2})dt$

(b) $F(r(t)) \cdot r'(t)dt = \cancel{(t^{-2}, -1)dt}$

$(t^{-2}-1)dt$

$\int_1^2 (t^{-2}-1)dt = [-t^{-1}-t]_1^2 = -\frac{1}{2}$

~~9.~~ $\cancel{r(t) = \langle t, t^3 \rangle}$

$\cancel{r(s) = \langle s, s^3 \rangle}$

$\cancel{r'(s)ds = \langle 1, 3s^2 \rangle ds}$

$\cancel{f(r(s)) = f(s, s^3) = \ldots}$

$\int_0^1$

9. $r(s) = \langle s, s^3 \rangle$

$r'(s)ds = \langle 1, 3s^2 \rangle ds$

$f(r(s)) = \cancel{\sqrt{9t^4+1}}\ \sqrt{9s^4+1}$

$f(r(s)) \cdot r'(s)ds = (\sqrt{9s^4+1})^2 = (9s^4+1)ds$

$\int_0^1 (9s^4+1)ds = [\frac{9s^5}{5}+s]_0^1 = \frac{14}{5}$

11. $r'(t)dt = (2, 3, 4)dt$

$f(r(t)) = 16t^2$

$f(r(t)) \cdot r'(t)dt = 16\sqrt{29}t^2dt$

$\int_0^2 16\sqrt{29}t^2dt = [\frac{16}{3}\sqrt{29}t^3]_0^2$

$= \frac{128\sqrt{29}}{3}$

13. For the first path,

$r(t) = (0, 2t, 1-t)$

$f(r(t)) = 0$.

$\therefore$ the path is also 0.

For the second path

$r(t) = (t, 2-t, t)$.

$r'(t)dt = (1, -1, 1)dt = \cancel{\sqrt{3}}dt$.

$f(r(t)) = te^{t^2}$

$\sqrt{3}\int_0^1 te^{t^2}dt = \frac{\sqrt{3}}{2}[e^{t^2}]_0^1 = \frac{\sqrt{3}}{2}(e-1)$

17. $r(2) = (8, -6, 24)$.

$r(5) = (20, -15, 60)$.

$r'(t)dt = (4, -3, 12)dt$.

$\int_2^5 13dt = 39$.

It represents the distance between $(8, -6, 24)$ and $(20, -15, 60)$

27. $dy = 2xdx$.

$\int_0^2 (x^2 - 2x^2)dx$

$= [-\frac{x^3}{3}]_0^2$

$= -\frac{8}{3}$

29. $x = t$, $y = 4t$, $z = 4t$.

$dx = dt$, $dy = 4dt$, $z = 4dt$.

$\int_0^1 -3tdt + 0 + 16tdt$

$= \int_0^1 13tdt$

$= [\frac{13}{2}t^2]_0^1$

$= \frac{13}{2}$

31. $\cancel{x=1-t,\ y=t.}$

$\cancel{dx=-dt,\ dy=dt}$

$\cancel{\int_0^1 \frac{t\,dt+(1-t)dt}{2t^2-2t+1}}$

$= \quad y=1-x.$

$dy=-dx.$

$\int_0^1 -\frac{1}{x^2+y^2}\,dx$

$= \cancel{\int \left[\frac{\arctan(\frac{1}{y})}{y}\right]}$

$=\left[\arctan(1-2x)\right]_0^1$

$=-\frac{\pi}{4}-\frac{\pi}{4}$

$=-\frac{\pi}{2}$

35. The first path:

$r(t)=(0,0,t).$

$r'(t)dt=(0,0,1)\,dt.$

$F(r(t))=(e^t,1,1)$

$\int_0^1 1\,dt=1.$

The second path

$r(t)=(0,t,1)$

$r'(t)dt=(0,1,0)\,dt.$

$F(r(t))=\cancel{(1,e^{-1},e)}\ (e,e^{-t},e^t).$

$\cancel{\int_0^1 e^{-1}dt=e^{-1}}.$

$\int_0^1 e^{-t}dt=[e^{-t}]_0^1=-e^{-1}+1.$

The third path

$r(t)=(-t,1,1).$

$r'(t)dt=(-1,0,0)\,dt.$

$F(r(t))=(e,e^{-t-1},e).$

$-\int_0^1 e\,dt=-e.$

The total path is the sum of them, it equals to

$(2-e-e^{-1}).$

16.3.

1. $\because F=\nabla f.$

$\therefore f$ is conservative

$\int_C F\cdot dr=f(1,1,\pi)-f(0,0,0)$

$=0-0$

$=0.$

3. $\frac{d}{dx}3x=3\,dx$

$\frac{d}{dy}3y^2=6y\,dy$

$\nabla f=(3,6y)=F.$

$f(r(4))-f(r(1))$

$=(3\times4+3\times\frac{1}{4})-(3+12)$

$=-\frac{9}{4}.$

5. $\frac{d}{dx}xye^z=ye^z\,dx.$

$\frac{d}{dy}xye^z=xe^z\,dy$

$\frac{d}{dz}xye^z=xye^z\,dz$

$\therefore F(x,y,z)=\nabla f$

$f(r(2)) - f(r(1))$

$= 32e - 1$.

9. $F_1 = y^2$

$F_2 = 2xy + e^z$

$F_3 = ye^z$.

$\frac{dF_1}{dy} = 2y$

$\frac{dF_2}{x} = 2y$

They're the same.

$\frac{dF_2}{dz} = e^z$

$\frac{dF_3}{dy} = e^z$

They're the same.

$\frac{dF_1}{dz} = 0$

$\frac{dF_3}{dx} = 0$

They're the same.

So it is conservative.

or x, $f = xy^2 + g(y,z)$

For y, $f = xy^2 + ye^z + h(z)$.

For z, $f = ye^z$.

$\therefore f = xy^2 + ye^z$.

13. $F_1 = z\sec^2 x$

$F_2 = z$

$F_3 = y + \tan x$.

$\frac{dF_1}{dy} = 0$

$\frac{dF_2}{dx} = 0$

They're the same

$\frac{dF_2}{dz} = 1$

$\frac{dF_3}{dy} = 1$

They're the same

$\frac{dF_1}{dz} = \sec^2 x$

$\frac{dF_3}{dx} = \sec^2 x$

They're the same

$\therefore$ it is conservative.

For x, $f = z\tan x + g(y,z)$.

For y, $f = yz + z\tan x + h(z)$.

For z, $f = yz + z\tan x$

$\therefore f = yz + \tan x$

15. $F_1 = 2xy + 5$.

$F_2 = x^2 - 4z$.

$F_3 = -4y$

$\frac{dF_1}{dy} = 2x$.

$\frac{dF_2}{dx} = 2x$

They're the same

$\frac{dF_2}{dz} = -4$.

$\frac{dF_3}{dy} = -4$

They're the same.

$\frac{dF_1}{dz} = 0$

$\frac{dF_3}{dx} = 0$

> They're the same.

$\therefore$ it is conservative.

For x, $f = x^2y + 5x + g(y, z)$.

For y, $f = x^2y - 4yz + 5x + h(z)$.

For z, $f = x^2y - 4yz + 5x$.

$\therefore F = x^2y - 4yz + 5x$.

17. $F_1 = 2xyz$

$F_2 = x^2z$

$F_3 = x^2y$

$\frac{dF_1}{dy} = 2xz$

$\frac{dF_2}{dx} = 2xz$

> They're the same.

$\frac{dF_2}{dz} = x^2$

$\frac{dF_3}{dy} = x^2$

> They're the same.

$\frac{dF_1}{dz} = 2xy$

$\frac{dF_3}{dx} = 2xy$

> They're the same.

$\therefore$ it is conservative

For x, $f = x^2yz + g(y, z)$

For y, $f = x^2yz + {}^{0}h(z)$.

For z, $f = x^2yz$

$\therefore f = x^2yz$

$f(r(2)) - f(r(0)) = 16 - 0 = 16$

19. $\because F = \nabla f$.

$\therefore$ It is conservative

For r_1,

$f(r_1(1)) - f(r_1(0)) = 1 - 0 = 1$.

For r_2,

$f(r_2(1)) - f(r_2(0)) = 1 - 0 = 1$.

$\therefore$ They're the same.