homework 10

16.2

3.) $F = \langle y^2, x^2 \rangle$ $r(t) = (t, t^{-1})$

a.) $\langle t^{-2}, t^2 \rangle$, $dr = 1, -1t^{-2}$ dt

b.) $t^{-2} \cdot 1 + t^2(-t^{-2})$

$t^{-2} - 1 \to \int_1^2 (t^{-1} - 1)dt = -\frac{1}{2}$

9.) $f(x,y) = \sqrt{1+9xy}$

$y = x^3$, for $0 \le x \le 1$

$x = t$, $y = t^3$

$c(t) = \langle t, t^3 \rangle$, $c'(t) = \langle 1, 3t^2 \rangle dt$

$\sqrt{1+9t^4} \to \int_0^1 \sqrt{1+9t^4} \cdot \sqrt{1+9t^4} = \int_0^1 1+9t^4 \to 1+\frac{9}{5}t^5 \to 1+9/5 = 2.8$

11.) $f(x,y,z) = z^2$

$(2t)^2 + (3t)^2 + (4t)^2 = 4t^2 + 9t^2 + 16t^2 = \sqrt{29t^2} \to \sqrt{29}\,t$

$\int z^2 \to \frac{z^3}{3} \to \frac{128\sqrt{29}}{3}$

13.) $f(x,y,z) = xe^{z^2}$

$c_1(t) = \frac{d}{dt}(0, 2t, 1-t) = (0, 2, -1)$ $|c_1| = \sqrt{5}$ $0 \cdot e^{(1-t)^2} = 0$

$c_2 \; \frac{d}{dt} \langle t, -t, t \rangle = \langle 1, -1, 1 \rangle \to |c_2| = \sqrt{3}$

$\int c_2 t = xe^{z^2} = te^{t^2} \to \int_{c_2} \to \int_0^1 xe^{t^2}\sqrt{3} \to \int_0^1 \frac{\sqrt{3}}{2}e^{t^2} \to$

$\frac{\sqrt{3}}{2}(e-1)$

17.) $\int_c 1\,ds$ $r(t) = (4t, -3t, 12t)$ for $2 \le t \le 5$

$\sqrt{16t^2 + 9t^2 + 144t^2} = \sqrt{169t^2}$ $13t$

$\int_c 1 \to \int_2^5 \|c\| \to \int_2^5 13t \to 13(3) = 39$

$(8, -6, 24)$ to $(20, -15, 60)$

27.) $\int_C y\,dx - x\,dy \qquad y = x^2 \quad 0 \le x \le 2$

$t = x \qquad t^2 = y \qquad dy = 2x\,dx$

$\int_C t^2 dt - t\,dt$

$t^2 \to \frac{t^3}{3} \to \frac{2^3}{3} = \frac{8}{3}$

29.) $\int (x-y)\,dx + (y-z)\,dy + z\,dz \qquad (0,0,0) \qquad (1,4,4)$

$\frac{x-0}{1-0} = \frac{y-0}{4-1} = \frac{z-0}{4-1} = t \;:\; x = \frac{y}{3} = \frac{z}{3} = t$

$x = t,\ y = 3t,\ z = 3t$

$dx = dt \quad dy = 3dt \quad z = 3dt$

$(t - 3t)\,dt + (3t - 3t)\,dy + 3(3t)\,dt \to -2t\,dt + 9t\,dt = 7t\,dt$

$\int_0^1 \to \frac{13}{2}$

31.) $\int \frac{-y\,dx + x\,dy}{x^2+y^2} \qquad (1,0) \quad (0,1)$

$\frac{x-1}{0-1} = \frac{y-0}{1-0} = \frac{x-1}{-1} = \frac{y}{1} = t$

$x - 1 = -t \to x = -t+1 \quad y = t \qquad dx = -dt \qquad dy = dt$

$\frac{-t(-dt) + (-t+1)(dt)}{(-t+1)^2 + (t^2)} = \frac{t\,dt + (-t+1)dt}{t^2 - 2t + 1 + t^2}$

$\frac{t\,dt - t\,dt + dt}{2t^2 - 2t + 1} = \frac{dt}{t^2 + (1-t^2)}$

$\frac{1}{t^2 + (1-t^2)}\,dt \to \int \frac{1}{u^2+1} \to \arctan(2t-1)$

$u = 2t - 1$

$\int_0^1 \tan^{-1}(2t-1) = \boxed{\frac{\pi}{2}}$

35.) cannot tell b/c path...

No.3

1.) $f(x,y,z) = xy(\sin(yz))$

$\nabla f = y\sin(yz)$, $x(\sin(zy) + zy\cos(zy))$, $xy^2\cos(yz)$

0

3.) $F(x,y) = \langle 3, 6y \rangle \rightarrow f(x,y) = 3x + 3y^2$

$\nabla f = 3, 6y$

5.) $F(x,y,z) = ye^z i + xe^z j + xye^z k \quad f(x,y,z) = xye^z$

$\nabla f = ye^z, xe^z, xye^z$

9.) $F = y^2 i + (2xy + e^z) j + (ye^z) k$

$\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 0^{?} \rightarrow$ ~~Ryy~~ $e^z - 2y \rightarrow$ ~~not conservative~~

$\boxed{y^2x + e^z y}$

13.) $F = \langle z\sec^2 x, z, y + \tan x \rangle$

$0 - 0 = $ cons.

~~not conservative~~

$z\tan x + zy$

15.) $\langle 2xy + 5, x^2 - 4z, -4y \rangle$

$x^2y + 5x - 4zy$

17.) $\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz \qquad r(t) = t^2, \sin(\pi t/4), e^{t^2-2t}$

$(\pi\cos\pi)$ $\quad e^{t^2,2t}$ $\qquad dx = 2t\,dt \quad dy = \pi\cos\pi\,dt, \ e^{t^2}\cdot 2t\,dt$

$2(2(t^2)(\sin\pi t/4))(e^{t^2-2t}) + (t^2)^2(e^{t^2-2t}) + (t^2)^2(\sin(\pi t/4))$

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19.) $f = x^2y - z \quad r_1 = \langle t, t, 0 \rangle \quad 0 \leq t \leq 1$

$r_2 = \langle t, t^2, 0 \rangle \quad 0 \leq t \leq 1$

$\nabla f = \langle 2yx, x^2, \cancel{z}-1 \rangle$

$\int_{c_1} 2(t)(t)i + t^2j + 0 \rightarrow \int_0^1 2t^2 i + t^2 j = \frac{\cancel{4}t^3}{3} i + \frac{t^3}{3} = \frac{1}{3}i + \frac{1}{3}j$

$\int_{c_2} 2(t)(t^2)i + t^2 j \rightarrow \int_0^1 2t^3 + t^2 j = \frac{6t^4}{4} + \frac{t^3}{\cancel{3}} = \frac{6}{4} + \frac{1}{3}$

$\boxed{1}$