11/13/20. 16.2 Homework

16.2 # 3, 9, 11, 13, 17, 27, 29, 31, 35

3) $F = \langle y^2, x^2 \rangle$ C: $y = x^{-1}$ for $1 \le x \le 2$

a) $F(r(t)) = \langle t^{-2}, t^2 \rangle$ $r(t) = \langle t, t^{-1} \rangle$

$dr = \langle 1, -t^{-2} \rangle dt$

b) $\langle t^{-2}, t^2 \rangle \cdot \langle 1, -t^{-2} \rangle = t^{-2} - 1$

$\int_1^2 t^{-2} - 1\, dt = -\frac{1}{2}$

9) $f(x,y) = \sqrt{1+9xy}$ $y = x^3$ for $0 \le x \le 1$

$\int_0^1 \sqrt{1+9x^4} = \frac{14}{5}$

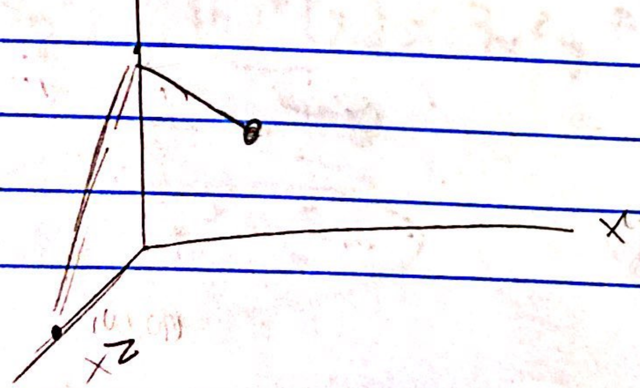
11) $f(x,y,z) = z^2$, $r(t) = (2t, 3t, 4t)$. $0 \le t \le 2$

$r'(t) = \langle 2,3,4 \rangle$, $\|r'(t)\| = \sqrt{4+9+16} = \sqrt{29}$

$f(r(t)) = 16t^2$

$\int_0^2 16t^2 \cdot \sqrt{29}\, dt = \frac{128\sqrt{29}}{3}$

13) $f(x,y,z) = xe^{z^2}$ from $(0,0,1)$ to $(0,2,0)$ to $(1,1,1)$



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$$\begin{array}{r} 144 \\ +25 \\ \hline 169 \end{array}$$

17) $\int 1\, ds$, C is parameterized by $r(t) = (4t, -3t, 12t)$

$r'(t) = \langle 4, -3, 12 \rangle$, $\|r'(t)\| = 13$

$\int_2^5 1 \cdot 13\, dt = 39$

27) $\int_C y\,dx - x\,dy$, $y = x^2$ from $0 \le x \le 2$.

$dy = 2x\, dx$.

$\int x^2 dx - x \cdot 2x\, dx = -\frac{8}{3}$.

29) $\int (x-y)dy + (y-z)\, dy + z\, dz$ from $(0,0,0)$ to $(1,4,4)$

$r(t) = \langle t, 4t, 4t \rangle$

$r'(t) = \langle 1, 4, 4 \rangle$

$\|r'(t)\| = \sqrt{33}$

$F(r(t)) = \langle x-y, y-z, z \rangle$

$\langle t-4t, 0, 4t \rangle$.

$\langle -3t, 0, 4t \rangle \cdot \langle 1, 4, 4 \rangle = 13t$.

$\int_0^1 13t\, dt = \frac{13}{2}$.

31) $\int_C \frac{-y\,dx + x\,dy}{x^2+y^2}$ from $(1,0)$ to $(0,1)$

$r(t) = (1-t)\langle 1, 0 \rangle + t\langle 0, 1 \rangle$ $r'(t) = \langle 1, 1 \rangle$

$\int_0^1 \frac{t}{(1-t)^2+t^2} + \frac{1-t}{(1-t)^2+t^2} dt = \frac{\pi}{2}$

e^{-t} $\frac{-2e^{-t}}{2}$ $\frac{e^{-2t}}{2}$

35) $F(x,y,z) = \langle e^{z}, e^{x-y}, e^{y} \rangle$

$\langle 0,0,1 \rangle$ $(0,0,0)$ to $(0,0,1) = (1-t)\langle 0,0,0 \rangle + t\langle 0,0,1 \rangle = \langle 0,$

$\langle 0,1,0 \rangle$ $(0,0,1)$ to $(0,1,1) = \langle 0,t,1 \rangle$.

$\langle -1,0,0 \rangle$ $(0,1,1) \to (-1,1,1) = \langle t,1,1 \rangle$

derivative

$\int F \cdot ds = \int F_1 ds + \int F_2 ds + \int F_3 ds.$

$\int_0^1 (0+0+1)\,dt = 1$

$\int_0^1 (e^{-t})\,dt = \frac{1}{2e^2}$

$\int_0^1 e\,dt = e$

$1 + \frac{1}{2e^2} + e$

4/20 16.3 Conservative Vector Fields.

16.3 # 1, 3, 5, | 9, 13, 15, 17.

1) $f(x,y,z) = xy\sin(yz)$

$\nabla f = \langle y\sin(yz), +x(\sin(yz) + yz\cos yz), xyz\cos yz \rangle$

$F = \langle y\sin yz, x(\sin(yz) + yz\cos yz), xyz\cos yz \rangle$

$\int_0^0 F\, dr = 0.$

3) $F(x,y) = \langle 3, 6y \rangle$ $f(x,y) = 3x + 3y^2$, $r(t) = \langle t, 2t^{-1} \rangle$.

$F(r(t)) = \langle 3, \frac{12}{t} \rangle$ $r'(t) = \langle 1, -2t^{-2} \rangle$

$\int F(r(t))\, r'(t) = \int_1^4 \langle 3, \frac{12}{t} \rangle \cdot \langle 1, -2t^{-2} \rangle\, dt$

$\int_1^4 (3 - \frac{24}{t^3})\, dt = -\frac{9}{4}$

5) $F(x,y,z) = \langle ye^z, xe^z, xye^z \rangle$

$f(x,y,z) = xye^z.$

$r(t) = \langle t^2, t^3, t-1 \rangle$ from $[1, 2]$.

$r'(t) = \langle 2t, 3t^2, 1 \rangle.$

$F(r(t)) = \langle 3t^3 e, 2te, 6t^3 e \rangle$

$\int F(r(t)) \cdot r'(t)\, dt = \int 6t^3 e + 6t^3 e + 6t^3 e = \frac{135e}{2}$

9) $F = \langle y^2, (2xy + e^z), ye^z \rangle$
$\text{curl}(F) = \langle 0,0,0 \rangle \rightarrow$ This field is conservative.

$f = \int y^2 dx = xy^2 + h(y,z)$
$f_y = 2xy + h_y$
$f = xy^2 + ye^z + g(z)$
$f_z = ye^z$
$g(z) = 0$
$f = xy^2 + ye^z$.

13) $F = \langle z\sec^2 x, z, y + \tan x \rangle$.
$\text{curl}(F) = \langle 0,0,0 \rangle \rightarrow$ Conservative

$f = \int f\, dx = z\tan x + h(y,z)$
$f_y = h_y \quad h_y = z$
$f = z\tan x + yz + g(z)$
$f = z\tan x + yz$.

15) $F = \langle 2xy + 5, x^2 - 4z, -4y \rangle$
$f_x = 2xy + 5, \; f_y = x^2 - 4z, \; f_z = -4y$
$f = \int f_x\, dx = x^2y + 5x + h(y,z)$
$f_y = x^2 + h_y, \; h_y = -4z$
$f = x^2 - 4yz + g(z)$
$f = x^2 - 4yz$

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1) $\int 2xyz\, dx + x^2z\, dy + x^2y\, dz$. $r(t) = \langle t^2, \sin(\frac{\pi t}{4}), e^{t^2-2t} \rangle$

$r'(t) = \langle 2t, \frac{\pi \cos(\pi t}{4}, (2t-2)e^{t^2-2t} \rangle$

$F(r(t)) = 2t^2 \sin(\frac{\pi t}{4}) (e^{t^2-2t})\, i$

$+ t^4 (e^{t^2-2t})\, j$

$+ (t^4) \sin(\frac{\pi t}{4}))$

$= \int_0^2 F(r(t)) \cdot r'(t)\, dt\ 16$