16.2 [3, 9, 11, 13, 17, 27, 29, 31, 35]
16.3 [1, 3, 5, 9, 13, 17, 19]

16.2
3) $F = \langle y^2, x^2 \rangle$; $C: y = x^{-1}$ for $1 \le x \le 2$, oriented from left to right

a) $F(r(t))$, $dr = r'(t)$ for $r(t) = \langle t, t^{-1} \rangle$

$x = t \quad y = t^{-1}$

$F(r(t)) = \langle t, t^{-1} \rangle \quad dr = \langle dt, -t^{-2} dt \rangle$

$F(r(t)) \cdot dr = (t^{-2} - 1) dt$

$$\int_1^2 t^{-2} - 1 \, dt = \frac{-1}{2}$$

9) $f(x,y) = \sqrt{1+9xy}$, $y = x^3$ for $0 \le x \le 1$

$r(t) = \langle t, t^3 \rangle \rightarrow \| r'(t) \| = \sqrt{1+9t^4}$
$x = t \quad y = t^3$

$$\int_0^1 \sqrt{1+9t^4} \cdot \sqrt{1+9t^4} \, dt = 2.8$$ * Use Maple

11) $f(x,y,z) = z^2$, $r(t) = \langle 2t, 3t, 4t \rangle$, $0 \le t \le 2$

$x = 2t$
$y = 3t$
$z = 4t$
$f(r(t)) = 16t^2$
$ds = \| r'(t) \| dt = \sqrt{29} \, dt$

$$\rightarrow \int_0^2 16t^2 \sqrt{29} \, dt = \frac{128\sqrt{29}}{3}$$ * use maple

13) $f(x,y,z) = xe^{z^2}$, Point A: $(0,0,1)$, Point B: $(0,2,0)$, Point C: $(1,1,1)$

1 $\int_{AB} f(r(t)) \cdot ds$ $\{ r(t) = \langle 0,0,1 \rangle + t \langle 0,2,0 \rangle = \langle 0, 2t, 1 \rangle \{ \| r'(t) \| = 2$
$x = 0 \quad y = 2t ; y' = 2dt \quad z = 1 \quad dz = 0$

$$\int_0^1 0 e^{z^2} dt = 0$$

2 $\int_{BC} f(r(t)) \cdot ds$ $\{ r(t) = \langle 0,2,0 \rangle + t \langle 1,1,1 \rangle = \langle t, t+2, t \rangle \Rightarrow | r'(t) | = \sqrt{3}$
$x = t \quad dx = dt \quad y = t+2 \quad dy = dt \quad z = t \quad dz = dt$

$$\int_0^1 \sqrt{3} t e^{t^2} dt = \frac{\sqrt{3}}{2} e^u \Big|_0^1 = \frac{\sqrt{3}}{2} e - \frac{\sqrt{3}}{2} =$$

$u = t^2 \Rightarrow [0,1]$

$\frac{du}{2} = t\,dt$

17) $\int_C 1\,ds$; $C: r(t) = \langle 4t, -3t, 12t \rangle$ $t=2$ and $t=5$

$\|r'(t)\| = \sqrt{4^2+3^2+144^2}$

$=$

$\int_2^5 1\; 13\,dt = 39$; The integral represents the distance between the points $(8,-6,24)$, $(20,-15,60)$

27) $\int_C y\,dx - x\,dy$ parabola $y=x^2$ for $0 \le x \le 2$

$r(t) = \langle t, t^2 \rangle$

$x = t$ $y = t^2$

$dx = dt$ $dy = 2t\,dt$

$\int_0^2 t^2\,dt - t(2t\,dt) = -\frac{8}{3}$

29) $\int_C (x-y)\,dx + (y-z)\,dy + z\,dz$, line segment $(0,0,0)$ to $(1,4,4)$

$r(t) = t\langle 1,4,4\rangle = \langle t, 4t, 4t\rangle$

$x=t$ $dx = dt$ $y = 4t$ $dy = 4dt$ $z = 4t$ $dz = 4dt$

$\int_0^1 (t-4t)\,dt + \cancel{(4t-4t)}(4dt) + 4t(4dt) = \frac{13}{2}$ * use maple

31) $\int_C \frac{-y\,dx + x\,dy}{x^2+y^2}$ line segment $(1,0)$ to $(0,1)$

$r(t) = \langle 1,0\rangle + t\langle 0,1\rangle = \langle 1, t\rangle$

$x=1$ $dx=0$ $dy = dt$ $y=t$

$\int_0^1 \frac{-t(0) + dt}{1+t^2} = \frac{\pi}{2}$ * use maple

35) $\int_{PA} f(x,y,z)\,ds + \int_{AB} f(x,y,z)\,ds + \int_{BQ} F(x,y,z)\,ds$ $F(x,y,z) = e^z, e^{x-y}, e^y$

1) $\int_{PA} \langle e, 1, 1\rangle \cdot \langle 0, 0, dt\rangle = 1$

$r_{PA}(t) = \langle 0,0,t\rangle$

$\|r_{PA}\| = 1$

2) $\int_{AB} \langle e^{t+1}, e^{-t}, e^t\rangle \cdot \langle 0,1,1\rangle dt = \int_0^1 e^{-t} + e^t\,dt = e^{-1}(e^2-1)$

$r_{AB} = \langle 0,0,1\rangle + t\langle 0,1,1\rangle = \langle 0, t, t+1\rangle \rightarrow \langle 0,1,1\rangle dt$

$x=0$ $y=t$ $z=t+1$

③ $\int_{BA} \langle e^{t+1}, e^{-2t+1}, e^{t+1} \rangle \cdot \langle -1, 1, 1 \rangle dt = \frac{1}{e}$

$r_{BA} = \langle 0, 1, 1 \rangle + t\langle -1, 1, 1 \rangle = \langle -t, t+1, t+1 \rangle$

$x = -t,\ y = t+1,\ z = t+1$

$2 - e - \frac{1}{e}$

16.3 [1, 3, 5, 9, 13, 17, 19]

1) $f(x,y,z) = xy\sin(z)$ $\quad C: (0,0,0)$ to $(1,1,\pi)$

$f(1,1,\pi) - f(0,0,0) = 0$ because F is conservative

3) $F(x,y) = \langle 3, 6y \rangle$, $f(x,y) = 3x + 3y^2$; $r(t) = \langle t, 2t^{-1} \rangle$; $1 \le t \le 4$

$r_0 = \langle 1, 2 \rangle$

$r_f = \langle 4, 1/2 \rangle$

$\nabla f = F$; $\nabla f = \langle 3, 6y \rangle$

$f(4, 1/2) - f(1,2) = -\frac{9}{4}$

5) $F(x,y) = ye^z i + xe^z j + xye^z k$, $f(x,y,z) = xye^z$; $r(t) = \langle t^2, t^3, t-1 \rangle$; for $1 \le t \le 2$

$\nabla f = \langle ye^z, xe^z, xye^z \rangle$

$r_0 = \langle 1, 1, 0 \rangle$ $\quad r_f = \langle 4, 8, 1 \rangle$

$f(4,8,1) - f(1,1,0) = 32e - 1$

9) $F = y^2 i + (2xy + e^z) j + ye^z k$

$\int y^2\, dx = xy^2 + g(y,z)$ $\qquad xy^2 + ye^z$

$\int (2xy + e^z)\, dy = xy^2 + ye^z + g(x,z)$

$\int ye^z\, dz = ye^z + g(x,y)$

13) $F = \langle z\sec^2 x, z, y + \tan x \rangle$

$\text{curl}(F) = \left\langle \frac{\delta}{\delta x}, \frac{\delta}{\delta y}, \frac{\delta}{\delta z} \right\rangle \times \langle z\sec^2 x, z, y + \tan(x) \rangle = 0$ $\quad$ * used coding

$\int z\sec^2 x\, dx = z\tan(x) + g(y,z)$ $\qquad yz + z\tan(x)$

$\int z\, dy = yz + g(x,z)$

$\int (y + \tan x)\, dz = yz + z\tan(x) + g(x,y)$

17) $\int_C 2xyz\, dx + x^2 z\, dy + x^2 y\, dz$

over the path $r(t) = \langle t^2, \sin(\pi t/4), e^{t^2-2t} \rangle$ for $0 \le t \le 2$

$r(t) = t^2, \sin(\pi t/4), e^{t^2-2t}$

$r'(t) = \langle 2t, \frac{\pi}{4}\cos(\pi t/4), e^{t^2-2t}(2t-2) \rangle \, dt$

* substitute x, y, z with parameters from $r(t)$
dx, dy, dz with parameters from $r'(t)$

* Used Maple: 16

19) $f = x^2y - z$, $r_1 = \langle t, t, 0 \rangle$. for $0 \le t \le 1$ and $r_2 = \langle t, t^2, 0 \rangle$ for $0 \le t \le 1$

$$\int_C F \cdot dr = f(r_1(1)) - f(r_1(0)) = 1$$

or

$$\int_C F \cdot dr = f(r_2(1)) - f(r_2(0)) = 1$$