16.2

3. $F=\langle y^2, x^2\rangle$, $y=x^{-1}$ for $1\le x\le 2$

(a).

9. $f(x,y)=\sqrt{1+9xy}$, $y=x^3$ for $0\le x\le 1$

$f(x)=\sqrt{1+9x^4}$

$ds=\sqrt{1+(\frac{dy}{dx})^2}\,dx$

$=\sqrt{1+(x^2)^2}\,dx$

$=\sqrt{1+x^4}\,dx$

$\int_0^1 \sqrt{(1+9x^4)(1+x^4)}\,dx$

$=1.7588$

11. $f(x,y,z)=z^2$, $r(t)=(2t,3t,4t)$ for $0\le t\le 2$

$r'(t)=(2,3,4)$

$|r'(t)|=\sqrt{2^2+3^2+4^2}=\sqrt{29}$

$\int_0^2 (4t)^2\cdot\sqrt{29}\,dt$

$=\frac{128\sqrt{29}}{3}$

13. $f(x,y,z)=xe^{z^2}$ from $(0,0,1)$ to $(0,2,0)$ to $(1,1,1)$

$C_1:(x(t),y(t),z(t))=(0,0,1)(1-t)+t(0,2,0)=(0,2t,1-t)$

$f(C_1(t))=(0)e^{(1-t)^2}=0$

$|C_1(t)|=\sqrt{0+4+1}=\sqrt{5}$

$\int_{C_1} xe^{z^2}ds=0$

$C_2:(0,2,0)(1-t)+t(1,1,1)$

$(x(t),y(t),z(t))=(t,2-t,t)$

$|C_2|=\sqrt{3}$ $\quad \int_{C_2} xe^{z^2}ds=\int_0^1(te^{t^2}\cdot\sqrt{3})dt=\frac{\sqrt{3}}{2}(e-1)$

$\int_C xe^{z^2}=\frac{\sqrt{3}}{2}(e-1)$

17. $r(t)=(4t,-3t,12t)$ for $2\leq t\leq 5$

$r'(t)=(4,-3,12)$

$|r'(t)|=\sqrt{4^2+(-3)^2+12^2}=13$

$\int_2^5 13\,dt=39$ $\quad r(2)=(8,-6,24)$

$r(5)=(20,-15,60)$

27. $\int_C y\,dx-x\,dy$, parabola $y=x^2$ $0\leq x\leq 2$

$y=x^2 \Rightarrow dy=2x\,dx$

$\int_0^2 x^2dx - x\cdot 2x\,dx$

$\int_0^2 (x^2-(2x^2))\,dx$

$=-\frac{x^3}{3}\Big|_0^2=-\frac{8}{3}$

29. $\int_C (x-y)\,dx+(y-z)\,dy+z\,dz$, from $(0,0,0)$ to $(1,4,4)$

$r(t)=\langle t,4t,4t\rangle$

$r'(t)=(1,4,4)$

$|r'(t)|=\sqrt{1+4^2+4^2}=\sqrt{33}$

$\int_0^1$

31. Let $a=(1,0)$ $b=\overrightarrow{AB}=(-1,1)$

$x=1-t$, $y=t$

$dx=-1\,dt$, $dy=dt \Rightarrow t=0$ and $t=1$

$\int_0^1 \frac{-t(-5)dt+(1-t)~~5dt~~)}{(1-t)^2+t^2}$

$=\frac{\pi}{2}$

35.

16.3

1. $f(x,y,z) = xy\sin(yz)$

$F = \nabla f = \langle y\sin(yz), x\sin(yz) + zxy\cos(yz), xy^2\cos(yz)\rangle$

$r(t) = \langle \pi, \pi, \pi t\rangle$

$F(r(t)) = \langle \pi\sin(\pi t^2), t\sin(\pi t^2) + \pi t^3(\cos(\pi t^2), t^3\cos\pi t^2\rangle$

$\int_c F dr = \frac{\pi - 1}{\pi^2}$

3. ~~$F(x,y,z) = ye^z i + xe^z j + xye^z k$ $f(x,y,z) = xye^z$~~

$F(x,y) = \langle 3, 6y\rangle$, $f(x,y) = 3x + 3y^2$; $r(t) = \langle t, 2t^{-1}\rangle$ for $1 \le t \le 4$

$r(1) = (1, 2)$, $r(4) = \langle 4, \frac{1}{2})$

$\int_c \nabla f \cdot dr = f(1,2) - f(4, \frac{1}{2})$

$= (3 + 12) - (12 + \frac{3}{4})$

$= 15 - 12\frac{3}{4}$

5. $F(x,y,z) = ye^z i + xe^z j + xye^z k$ $f(x,y,z) = xye^z$, $r(t) = (t^2, t^3, t-1)$ for $1 \le t \le 2$

$f_x = ye^z$

$f = xye^z + xye^z + g(y,z)$

$f_x = xe^z \Rightarrow xe^z + g_y = 2xy$

$f = xye^z + xye^z + \frac{xye^z}{z}$

$r(1) = (1, 1, 0)$ $r(2) = (4, 8, 1)$

$f(1,1,0) - f(4,8,1) = 32e - 1$

9. $F = y^2 i + (2xy + e^z) j + ye^z k$

$P = y^2$, $P_y = 2y$, $P_z = 0$

$Q = 2xy + e^z$, $Q_x = 2y$, $Q_z = e^z$

$R = ye^z$, $R_x = 0$, $R_y = e^z$

curl $F = \langle 0, 0, 0\rangle$

$f(x,y,z) = \int y^2 dx + g(y,z)$

$= xy^2 + g(y,z)$

$\frac{d}{dy}(xy^2) + g_y = 2xy + e^z$

$g_y = e^z$

$g(y,z) = \int e^z dy + h(z) = ye^z + h(z)$

$f(x,y,z) = xy^2 + ye^z + h(z)$

$\frac{d}{dz}(xy^2 + ye^z) + h'(z) = ye^z$

$h'(z) = 0$

$f(x,y,z) = xy^2 + ye^z + C$

$P = 2\sec x, \quad P_y = 0, \quad P_z = \sec^2(x)$

$Q = z \qquad Q_x = 0, \ Q_z = 1$

$R = y + \tan x \qquad R_x = y + \tan x \quad R_y = y + \tan x$

$\text{curl} F = \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle = \langle 1 + \tan x - 1, \sec^2(x) - (1 + \tan x), 0 - 0$

$= \langle 0, 0, 0 \rangle$

$f(x, y, z) = z\tan x + zy$

15. $F = \langle 2xy + 5, x^2 - 4z, -4y \rangle$

$P = 2xy + 5, \quad P_y = 2x + 5, \quad P_z = 0$

$Q = x^2 - 4z, \quad Q_x = 2x - 4z, \quad Q_z = x^2 - 4$

$R = -4y, \quad R_x = 0, \quad R_y = -4$

$\text{curl} F = \langle 0, 0, 0 \rangle$

$f(x, y, z) = x^2y + 5x - 4zy$

17. $\int_C 2xyz\, dx + x^2z\, dy + x^2y\, dz \qquad r(t) = (t^2, \sin(\pi t/4), e^{t^2 - 2t})$