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16.2 : 3, 9, 11, 13, 17, 27, 29, 31, 35

#3 $F = \langle y^2, x^2 \rangle$ curve $y = x^{-1}$ for $1 \le x \le 2$

a) $F(r(t))$ and $dr = r'(t)$ for the parametrization C given by $r(t) = (t, t^{-1})$

$F(r(t)) \longrightarrow \langle (t^{-1})^2, t^2 \rangle = \boxed{\langle t^{-2}, t^2 \rangle}$

$dr = r'(t)\,dt \longrightarrow r'(t) = \boxed{(1, -t^{-2})\,dt}$

b) $\langle t^{-2}, t^2 \rangle \cdot \langle 1, -t^{-2} \rangle = t^{-2}(1) + t^2(-t^{-2}) = t^{-2} - t^0 = \boxed{t^{-2} - 1}$

$\int_C F \cdot dr = \int_1^2 t^{-2} - 1 \; dt = -t^{-1} - t \Big|_1^2 = \left(-\frac{1}{2} - 2\right) - (-1 - 1) = \boxed{-\frac{1}{2}}$

$-\frac{5}{2} + \frac{4}{2}$

#9 $f(x,y) = \sqrt{1 + 9xy}$, $y = x^3$ for $0 \le x \le 1$

$y = x^3 \longrightarrow \langle t, t^3 \rangle$ $r'(t) = \langle 1, 3t^2 \rangle$ $\|r'(t)\| = \sqrt{1^2 + (3t^2)^2} = \sqrt{1 + 9t^4}$

$F(r(t)) = \sqrt{1 + 9(t)(t^3)} = \sqrt{1 + 9t^4}$

$\int_C f\,ds = \int_0^1 \sqrt{1+9t^4} \cdot \sqrt{1+9t^4}\; dt = \int_0^1 1 + 9t^4 \; dt = t + \frac{9}{5}t^5 \Big|_0^1 = 1 + \frac{9}{5} = \boxed{\frac{14}{5}}$

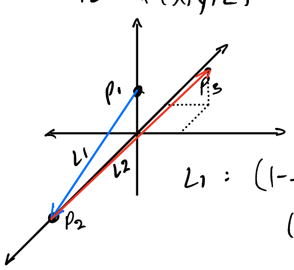
#11 $f(x,y,z) = z^2$ $r(t) = (2t, 3t, 4t)$ for $0 \le t \le 2$

$r'(t) = (2, 3, 4)$ $\|r'(t)\| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$

$f(r(t)) = (4t)^2 = 16t^2$

$\int_C f \cdot ds = \int_0^2 16t^2 \cdot \sqrt{29}\; dt = 16\sqrt{29} \cdot \frac{t^3}{3} \Big|_0^2 = \boxed{\frac{128\sqrt{29}}{3}}$

#13 $f(x,y,z) = xe^{z^2}$ piecewise linear path from $(0,0,1)$ to $(0,2,0)$ to $(1,1,1)$



$\int_C = \int_{L1} + \int_{L2}$

$L_1 : (1-t)(0,0,1) + t(0,2,0)$

$(0, 0, 1-t) + t(0,2,0)$

$\langle 0, 2t, 1-t \rangle$

$L_2 : (1-t)(0,2,0) + t(1,1,1)$

$(0, 2-2t, 0) + (t, t, t)$

$\langle t, 2-t, t \rangle$

$L_1'(t) = \langle 0, 2, -1 \rangle$ $\|L_1'(t)\| = \sqrt{4+1} = \sqrt{5}$

$L_2'(t) = \langle 1, -1, 1 \rangle$ $\|L_2'(t)\| = \sqrt{1+1+1} = \sqrt{3}$

$f(L_1(t)) = 0$ $f(L_2(t)) = t \cdot e^{t^2}$

$\int_0^1 0\,dt + \int_0^1 \sqrt{3} \cdot te^{t^2}\,dt = 0 + \sqrt{3} \cdot \frac{e^{t^2}}{2} \Big|_0^1$

$= \frac{\sqrt{3}}{2} \cdot e - \sqrt{3} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{3}}{2}(e-1)}$

#17 $\int_C 1\,ds$ where curve C parametrized by: $r(t) = (4t, -3t, 12t)$ for $2 \le t \le 5$

$r'(t) = \langle 4, -3, 12 \rangle \qquad \|r'(t)\| = \sqrt{16+9+144} = \sqrt{169} = 13$

$$\int_2^5 1 \cdot 13\,dt = 13t \Big|_2^5 = 65 - 26 = \boxed{39}$$

#27 $\int_C y\,dx - x\,dy$, parabola $y = x^2$ for $0 \le x \le 2$

$f = \langle y, -x \rangle$

$r(t) = \langle t, t^2 \rangle$

$r'(t) = \langle 1, 2t \rangle$

$f(r(t)) = \langle t^2, -t \rangle$

$$= \int_a^b F(r(t)) \cdot r'(t)\,dt$$

$$\langle t^2, -t \rangle \cdot \langle 1, 2t \rangle = t^2 + (-t)(2t) = t^2 - 2t^2 = -t^2$$

$$= \int_0^2 -t^2\,dt = -\frac{t^3}{3}\Big|_0^2 = \boxed{-\frac{8}{3}}$$

#29 $\int_C (x-y)\,dx + (y-z)\,dy + z\,dz$, line segment from $(0,0,0)$ to $(1,4,4)$

$f(x,y,z) = \langle x-y, y-z, z \rangle$

$r(t) = (1-t)(0,0,0) + t(1,4,4)$
$= (0,0,0) + (t, 4t, 4t)$
$= \langle t, 4t, 4t \rangle$

$r'(t) = \langle 1, 4, 4 \rangle$

$F(r(t)) = \langle t - 4t, 4t - 4t, 4t \rangle$
$= \langle -3t, 0, 4t \rangle$

$\langle -3t, 0, 4t \rangle \cdot \langle 1, 4, 4 \rangle = -3t + 0 + 16t$

$$\int_0^1 13t\,dt = \frac{13}{2}t^2 \Big|_0^1 = \boxed{\frac{13}{2}}$$

#31 $\int_C \frac{-y\,dx + x\,dy}{x^2+y^2}$ segment from $(1,0)$ to $(0,1)$

$f(x,y) = \left\langle -\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$

$r(t) = (1-t)(1,0) + t(0,1)$
$= (1-t, 0) + (0, t)$
$= (1-t, t)$

$r'(t) = \langle -1, 1 \rangle$

$F(r(t)) = \left\langle -\frac{t}{(1-t)^2+t^2}, \frac{1-t}{(1-t)^2+t^2} \right\rangle$

$F(r(t)) \cdot r'(t) = \frac{t}{(1-t)^2+t^2} + \frac{1-t}{(1-t)^2+t^2}$

$$\int_0^1 \frac{1}{(1-t)^2+t^2}\,dt =$$

#35 $F(x,y,z) = \langle e^z, e^{x-y}, e^y \rangle$ $P = (0,0,0)$ $Q = (-1,1,1)$

parametrization :

$$r(t) = (1-t)\langle 0,0,0\rangle + t\langle -1,1,1\rangle$$
$$= \langle 0,0,0\rangle + \langle -t,t,t\rangle$$
$$= \langle -t,t,t\rangle$$

$$r'(t) = \langle -1,1,1\rangle$$

$$F(r(t)) = \langle e^t, e^{-t-t}, e^t\rangle = \langle e^t, e^{-2t}, e^t\rangle$$

$$\langle e^t, e^{-2t}, e^t\rangle \cdot \langle -1,1,1\rangle = -e^t + e^{-2t} + e^t$$

$$\int_1^0 -e^t + e^{-2t} + e^t \, dt = -e^t + \frac{e^{-2t}}{-2} + e^t \Big|_0^1$$

$$= \left(-e - \frac{1}{2e^2} + e^1\right) - \left(-1 + \left(-\tfrac{1}{2}\right) + 1\right)$$

$$= -\frac{1}{2e^2} - \left(-\tfrac{1}{2}\right) = \boxed{\frac{1}{2} - \frac{1}{2e^2}}$$

16.3: 1, 3, 5, 9, 13, 15, 17, 19

#1 $f(x,y,z) = xy\sin(yz)$ $F = \nabla f$ $(0,0,0)$ to $(1,1,\pi)$

$$\int \nabla f \, ds = f(1,1,\pi) - f(0,0,0) = 1 \cdot 1 \cdot \sin(\pi) - 0 = \boxed{0}$$

#3 $F(x,y) = \langle 3, 6y\rangle$, $f(x,y) = 3x + 3y^2$, $r(t) = \langle t, 2t^{-1}\rangle$ for $1 \le t \le 4$

$$\nabla f(x,y) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle = \langle 3, 6y\rangle = F \qquad f(r(t)) = 3t + 3(2/t)^2 = 3t + 12/t^2$$

$$\int_1^4 F\,dr = f(4) - f(1) = \left(3(4) + \tfrac{12}{4^2}\right) - \left(3(1) + 12/1^2\right)$$
$$= \left(12 + \tfrac{3}{4}\right) - (3 + 12) = 12 + \tfrac{3}{4} - 15 = \boxed{-9/4}$$

#5. $F(x,y,z) = ye^z i + xe^z j + xye^z K$, $f(x,y,z) = xye^z$, $r(t) = \langle t^2, t^3, t-1\rangle$, $1 \le t \le 2$

$$\nabla f = \langle ye^z, xe^z, xye^z\rangle = F \checkmark \qquad f(r(t)) = t^2(t^3)e^{t-1} = t^5 e^{t-1}$$

$$\int_1^2 F\,ds = f(2) - f(1) = 2^5 e^{2-1} - 1^5 e^{1-1} = \boxed{32e - 1}$$

#9 $F = y^2 i + (2xy + e^z) j + ye^z k = \langle y^2, 2xy + e^z, ye^z \rangle$

$\frac{\partial F_1}{\partial y} = 2y \quad \frac{\partial F_2}{\partial x} = 2y \quad , \quad \frac{\partial F_2}{\partial z} = e^z = \frac{\partial F_3}{\partial y} = e^z \quad , \quad \frac{\partial F_3}{\partial x} = 0 = \frac{\partial F_1}{\partial z} = 0$

$\therefore$ conservative.

$f(x,y,z) = \int y^2 dx = y^2 x + g(y,z)$

$\frac{\partial}{\partial y}(y^2 x + g(y,z)) = 2xy + g_y(y,z) = 2xy + e^z$

$\therefore g_y(y,z) = e^z$

$g(y,z) = \int e^z dy = ye^z + h(z)$

$\frac{\partial}{\partial z}(y^2 x + ye^z + h(z)) = 0 + ye^z + h'(z) = ye^z$

$\therefore h'(z) = 0 \rightarrow h(z) = C$

$\boxed{\text{potential function: } f(x,y,z) = y^2 x + e^z + C}$

#13 $F = \langle z\sec^2 x, z, y + \tan x \rangle$

$\frac{\partial F_1}{\partial y} = 0 \quad \frac{\partial F_2}{\partial x} = 0 \quad , \quad \frac{\partial F_2}{\partial z} = 1 \quad \frac{\partial F_3}{\partial y} = 1 \quad , \quad \frac{\partial F_3}{x} = \sec^2 x = \frac{\partial F_1}{\partial z} = \sec^2 x$

$\therefore$ conservative.

$f(x,y,z) = \int z\sec^2 x \, dx = z\tan x + g(y,z)$

$\frac{\partial}{\partial y}(z\tan x + g(y,z)) = 0 + g_y(y,z) = z \quad \therefore g_y(y,z) = z$

$g(y,z) = \int z \, dy = zy + h(z)$

$\frac{\partial}{\partial z}(z\tan x + zy + h(z)) = \tan x + y + h'(z) = y + \tan(x) \quad \therefore h'(z) = 0$

$\hookrightarrow h(z) = C$

$\boxed{\text{potential fonction} = f(x,y,z) = z\tan x + zy + C}$

#15 $F = \langle 2xy+5,\ x^2-4z,\ -4y \rangle$

$$\frac{\partial F_1}{\partial y} = 2x = \frac{\partial F_2}{\partial x} = 2x, \quad \frac{\partial F_2}{\partial z} = -4 = \frac{\partial F_3}{\partial y} = -4, \quad \frac{\partial F_3}{\partial x} = 0 = \frac{\partial F_1}{\partial z} = 0$$

$\therefore$ conservative

$$f(x,y,z) = \int 2xy+5\ dx = yx^2+5x+g(y,z)$$

$$\frac{\partial}{\partial y}\left(yx^2+5x+g(y,z)\right) = x^2+0+g_y(y,z) = x^2-4z \quad \therefore g_y(y,z) = -4z$$

$$g(y,z) = \int -4z\ dy = -4zy+h(z)$$

$$\frac{\partial}{\partial z}\left(yx^2+5x-4zy+h(z)\right) = 0+0-4y+h'(z) = -4y \quad \therefore h'(z) = 0$$

$\hookrightarrow h(z) = C$

$$\boxed{\text{potential function: } f(x,y,z) = yx^2+5x-4zy+C}$$

#17 $\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz$ over $r(t) = (t^2, \sin(\pi t/4), e^{t^2-2t})$ for $0 \le t \le 2$

$f = \langle 2xyz,\ x^2z,\ x^2y \rangle$

$\nabla f = F = \langle 2yz, 0, 0 \rangle$

$a = (0,0,1) \quad b = (4,1,1)$

$$f(x,y,z) = \int 2xyz\,dx = x^2yz+g(y,z)$$

$$\frac{\partial}{\partial y}\left(x^2yz+g(y,z)\right) = x^2z+g_y(y,z) = x^2z \quad \therefore g_y(y,z) = 0 \rightarrow g(y,z) = C+h(z)$$

$$\frac{\partial}{\partial z}\left(x^2yz+C+h(z)\right) = x^2y+h'(z) = x^2y \quad \therefore h(z) = C$$

potential function at $C=0$: x^2yz

$$\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz = f(b)-f(a) = f(4,1,1)-f(0,0,1)$$
$$= 4^2(1)(1) - 0 = \boxed{16}$$

#19 $F = \nabla f \quad \int_C F\cdot dr$

$f = x^2y - z$, $r_1 = \langle t,t,0\rangle$ for $0 \le t \le 1$ and $r_2 = \langle t, t^2, 0\rangle$ for $0 \le t \le 1$

$r_1(0) = \langle 0,0,0\rangle \quad r_1(1) = \langle 1,1,0\rangle \qquad r_2(0) = (0,0,0) \quad r_2(1) = \langle 1,1,0\rangle$

$F = \nabla f = \langle 2xy,\ x^2,\ -1\rangle$

$$\int_C F\cdot dr = f(1,1,0) - f(0,0,0) = (2+1-1) - (0+0-1) = 3$$