Calc HW
Due 11/15

Rahul Paleja

Section 16.2 - # 3, 9, 11, 13, 17, 27, 29, 31, 35:

(3) Let $F = (y^2, x^2)$ and let C be the curve $y = x^{-1}$ for $1 \le x \le 2$ oriented from left to right

(a) Calculate $F(r(t))$ and $dr = r'(t)\,dt$ for the parameterization of C given by $r(t) = (t, t^{-1})$

$F(r(t)) = (t^{-2}, t^2)$

$dr = \langle 1, -t^{-2} \rangle dt$

(b) $\int F(r(t)) \cdot dr\, dt = \langle t^{-2}, t^2 \rangle \cdot \langle 1, -t^{-2} \rangle dt$

$= t^{-2} + (-t^0)$

$= t^{-2} - 1\ dt$

$\int_1^2 t^{-2} - 1\, dt = -t^{-1} - t\Big]_1^2$

$= (-(2)^{-1} - 2) - (-1^{-1} - 1)$

$= (-\frac{1}{2} - \frac{2}{1}) - (-2)$

$(-\frac{5}{2}) + (\frac{2}{1}) = \boxed{\frac{-1}{2}}$

(9) $f(x,y) = \sqrt{1+9xy}$ $\quad y = x^3$ for $0 \le x \le 1$

curve: $(x, x^3)$

Parameter t: curve $(t, t^3)$ $\quad 0 \le t \le 1$

$x(t) = t \quad y(t) = t^3 \quad x'(t) = 1 \quad y'(t) = 3t^2$

$ds = \sqrt{(1)^2 + (3t^2)^2}\, dt$

$= \sqrt{1 + 9t^4}\, dt$

$F(x,y) = \sqrt{1+9xy}$

$= \sqrt{1+9t^4}$

$\int_0^1 \sqrt{1+9t^4} \cdot \sqrt{1+9t^4}\, dt = \int_0^1 (1+9t^4)\, dt$

$= t + \frac{9t^5}{5}\Big]_0^1$

$= \frac{1}{1} + \frac{9}{5} = \boxed{\frac{14}{5}}$

(11) $f(x,y,z) = z^2 \quad r(t) = \langle 2t, 3t, 4t \rangle$ for $0 \le t \le 2$

$x'(t) = 2 \quad y'(t) = 3 \quad z'(t) = 4$

$ds = \sqrt{2^2 + 3^2 + 4^2}\, dt = \sqrt{29}\, dt$

$f(x,y,z) = (4t)^2 = 16t^2$

$16 \cdot \sqrt{29} \int_0^2 t^2 = \left.\frac{t^3}{3}\right]_0^2 = \frac{8}{3} \cdot 16\sqrt{29}$

$= \frac{128\sqrt{29}}{3}$

(13) $f(x,y,z) = xe^{z^2}$, piecewise linear path from $(0,0,1)$ to $(0,2,0)$ to $(1,1,1)$

2 part integral

$\langle 0,0,1 \rangle + t\langle 0,2,-1 \rangle \quad 0 \le t \le 1$

$= \langle 0, 2t, 1-t \rangle \quad x'(t) = 0,\ y'(t) = 2,\ z'(t) = -1$

$f(x,y,z) = 0 \quad ds = \sqrt{4+1} = \sqrt{5}\, dt$

$\int_0^1 0 \cdot \sqrt{5}\, dt = 0$ → Add this to next integral, geometrically area under this line + under next line

from $(0,2,0)$ to $(1,1,1)$

$= \langle 0,2,0 \rangle + t\langle 1,-1,1 \rangle = \langle t, 2-t, t \rangle \quad 0 \le t \le 1$

$ds = \sqrt{1^2 + (-1)^2 + (1)^2} = \sqrt{3}\, dt \quad x'(t) = 1 \quad y'(t) = -1 \quad z'(t) = 1$

$f(x,y,z) = te^{t^2}$

$\int_0^1 te^{t^2} \cdot \sqrt{3}\, dt = \sqrt{3}\int_0^1 te^{t^2}\, dt$

$u = t^2 \quad \frac{du}{2} = \frac{2t\,dt}{2}$

$= \frac{\sqrt{3}}{2}\int e^u\, du = \frac{\sqrt{3}}{2}\left[e^{t^2}\right]_0^1$

$= (e^1 - e^0)\frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{3}}{2}(e-1)}$

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Section 16.2 : # 17, 27, 29, 31, 35

(17) $x'(t) = 4 \quad y'(t) = -3 \quad z'(t) = 12$

$ds = \sqrt{4^2 + (-3)^2 + (12)^2}\, dt = \sqrt{16 + 9 + 144} = \sqrt{169} = 13\, dt$

$$\int_2^5 13\, dt = 13(5-2) = 13 \cdot 3 = 39 \rightarrow$$ Represents distance between points $(8,-6,24)$ & $(20,-15,60)$

$t = 2 \rightarrow r(2) = (8, -6, 24) \qquad r = 5 \rightarrow (20, -15, 60)$

(27) $\int_C y\,dx - x\,dy$ parabola $y = x^2$ for $0 \le x \le 2$

curve $(x, x^2)$

Parameter $t \rightarrow (t, t^2)$

$x'(t) = dt \quad y'(t) = 2t\, dt$

$ds = \sqrt{1^2 + (2t^2)} = \sqrt{4t^2 + 1}\, dt$

$$\int_0^2 t^2\, dt - t\, 2t\, dt$$

$$= \int_0^2 t^2\, dt - 2t^2\, dt = \frac{t^3}{3} - \frac{2t^3}{3}\Big]_0^2$$

$$= \boxed{\frac{-8}{3}}$$

(29) $\int_C (x-4)\,dx + (y-2)\,dy + z\,dz$ ; line segment from $(0,0,0)$ to $(1,4,4)$

$x = t \quad dx = dt \quad \langle 0,0,0 \rangle + t\langle 1,4,4 \rangle$

$y = 4t \quad dy = 4dt \quad = \langle t, 4t, 4t \rangle \quad 0 \le t \le 1$

$z = 4t \quad dz = 4dt$

$$\int_0^1 (t - 4t)\, dt + \cancel{(0)\,4dt} + 4t(4dt)$$

$$\int_0^1 13t\, dt = 13\int_0^1 t\,dt = 13\left(\frac{t^2}{2}\right]_0^1$$

$$= \boxed{\frac{13}{2}}$$

31 $\int_C \frac{-y\,dx + x\,dy}{x^2+y^2}$ segment from $(1,0)$ to $(0,1)$

$\langle 1,0\rangle + t\langle -1,1\rangle$

$= \langle 1-t, t\rangle \quad 0 \le t \le 1$

Ask in Recitation

$x = 1-t \quad dx = -dt \qquad y = t \quad dy = dt$

$$\int_0^1 \frac{-t(-dt) + (1-t)\,dt}{(1-t)^2 + t^2} = \int_0^1 \frac{1\,dt}{1-2t+2t^2}$$

$(1-t)(1-t)$

$1-2t+t^2$

$$= \int_0^1 \frac{1}{\left(\sqrt{2}t - \frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}}\,dt$$

$u = 2t-1 \quad \frac{du}{2} = \frac{2}{2}dt$

$\arctan(u)$

$$= \int \frac{1}{u^2+1}\,du$$

$$= \arctan(2t-1)\Big]_0^1 = \frac{\pi}{4} - \left(\frac{-\pi}{4}\right) = \frac{2\pi}{4} = \frac{\pi}{2}$$

$= \boxed{\frac{\pi}{2}}$

35 $F(x,y,z) = \langle e^z, e^{x-y}, e^y\rangle$

$0 \le t \le 1$

$\langle 0,0,0\rangle + t\langle 0,0,1\rangle = \langle 0,0,t\rangle$

$dx = 0 \quad dy = 0 \quad dz = dt$

$$\int_0^1 e^0\,dt = \int_0^1 dt = \boxed{1}$$

$0 \le t \le 1$

$(0,0,1)$ to $(0,1,1) \to \langle 0,0,1\rangle + t\langle 0,1,0\rangle$

$= \langle 0, t, 1\rangle \quad dx = 0 \quad dy = dt \quad dz = 0$

$$\int_0^1 e^{-t}\,dt = -e^{-t}\Big]_0^1 = -e^{-1} - (-e^0) = \boxed{-\frac{1}{e} + 1}$$

$0 \le t \le 1$

$(0,1,1)$ to $(-1,1,1) \quad \langle 0,1,1\rangle + t\langle -1,0,0\rangle = \langle -t, 1, 1\rangle$

$dx = -dt \quad dy = 0 \quad dz = 0$

$$\int_0^1 e^1 - dt = -e\int_0^1 dt = \boxed{-e}$$

Answer: $1 + \left(-\frac{1}{e} + 1\right) + (-e)$

$= \boxed{2 - \frac{1}{e} - e}$

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Rahul Palegar

16.3 - # 1, 3, 5, 9, 13, 15, 17, 19:

① Let $f(x,y,z) = xy\sin(yz)$ and $F = \nabla f$. Evaluate $\int_C F \cdot dr$ where $C$ is any path from $(0,0,0)$ to $(1,1,\pi)$

$$\int_C F\,dr = f(1,1,\pi) - f(0,0,0)$$
$$= \sin(\pi) - 0 = 0 - 0 = \boxed{0}$$

③ $F(x,y) = \langle 3, 6y\rangle$, $f(x,y) = 3x + 3y^2$, $r(t) = \langle t, 2t^{-1}\rangle$ for $1 \leq t \leq 4$

verify: $F = \nabla f$ $\quad f_x = 3 \quad f_y = 6y$
$\nabla f = \langle 3, 6y\rangle$

$r(4) = \langle 4, \frac{1}{2}\rangle$
$r(1) = \langle 1, 2\rangle$

$$\int_C F \cdot dr = f(Q) - f(P)$$
$$= f(4, \tfrac{1}{2}) - f(1,2)$$
$$= \left(\tfrac{4}{4}\right)\tfrac{12}{1} + \tfrac{3}{4} - (3 + 12)$$
$$= \tfrac{51}{4} - \tfrac{15}{1}\left(\tfrac{4}{4}\right) = \tfrac{51}{4} - \tfrac{60}{4} = \boxed{\tfrac{-9}{4}}$$

⑤ $F = \nabla f \quad \nabla f = \langle ye^z, xe^z, xye^z\rangle$
$$= ye^z i + (xe^z)j + (xye^z)k$$

$r(1) = \langle 1, 1, 0\rangle \quad r(2) = \langle 4, 8, 1\rangle$

$$f(4,8,1) - f(1,1,0) = \boxed{32e - 1}$$

(9) $F = \langle y^2, 2xy + e^z, ye^z \rangle$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^2 & 2xy+e^z & ye^z \end{vmatrix}$$

(1) $i(e^z - e^z) - j(0-0) + k(2y - 2y)$

$= i(0) - j(0) + k(0)$

$= \langle 0, 0, 0 \rangle \rightarrow F$ is conservative

(2) Find $f$ such that $F = \nabla f = \langle f_x, f_y, f_z \rangle$

$f_x = y^2 \quad f_y = 2xy + e^z \quad f_z = ye^z$

$f(x,y,z) = \int y^2\,dx = y^2x + g(y,z) \rightarrow$ differentiate to find $f_y$

Use $f_y = Q \quad f_y = 2xy + e^z \rightarrow \frac{d}{dy}\left(y^2x + g(y,z)\right) = 2xy + e^z$

$\cancel{2yx} + g_y(y,z) = \cancel{2xy} + e^z$

$-2xy \qquad -2xy$

$\int g_y(y,z) = \int e^z = e^z y + h(z)$

$f(x,y,z) \overset{d/dz}{=} \left(y^2x + ye^z + h(z)\right) = ye^z$

$0 + ye^z + h'(z) = ye^z \qquad \int h'(z) = 0\,dz$

$h(z) = 0 \rightarrow$ can ignore "+C"

$$\boxed{f(x,y,z) = y^2x + e^z y}$$

(13) $F = \langle z\sec^2 x, z, y + \tan x \rangle$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z\sec^2 x & z & y+\tan x \end{vmatrix}$$

$i(1-1) - j(\sec^2 x - \sec^2 x)$

$+ k(0-0)$

$= \langle 0,0,0 \rangle \rightarrow F$ is conservative

$f(x,y,z) = \int z\sec^2 x\,dx = z\tan x + g(y,z)$

$\partial/\partial y\left(z\tan(x) + g(y,z)\right) = z \rightarrow 0 + g_y(y,z) = \int z\,dy = zy + h(z)$

$f(x,y,z) = \left(z\tan x + zy + h(z)\right) \int d/dz$

$\cancel{\tan x + y} + h'(z) = \cancel{y + \tan x} \qquad \int h'(z) = 0\,dz = 0 \rightarrow$ No "+C" needed

$$\boxed{f(x,y,z) = z\tan x + zy}$$

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16.3 - #15,17,19:

(15) $F = \langle 2xy+5,\ x^2-4z,\ -4y\rangle$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy+5 & x^2-4z & -4y \end{vmatrix}$$

$i(-4-(-4)) - j(0-0) + k(2x-2x)$
$= 0i - 0j + 0k = \langle 0,0,0\rangle \Rightarrow F$ is conservative

$f(x,y,z) = \int 2xy+5\,dx = x^2y + 5x + g(y,z)$

$\cancel{x^2} + g_y(y,z) = \cancel{x^2} - 4z \qquad g(y,z) = -4z + h(z)$

$d/dz$

$f(x,y,z) = (x^2y + 5x - 4z + h(z))$

No "+c"

$-4 + h'(z) = \int(-4y+4)dz = -4yz + 4z$
$+4$

$f(x,y,z) = x^2y + 5x - \cancel{4z} - 4yz + \cancel{4z} = \boxed{x^2y + 5x - 4yz}$

(17) First Check if F is conservative: curl(F) = 0

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xyz & x^2z & x^2y \end{vmatrix}$$

$i(x^2-x^2) - j(2xy-2xy)$
$+ k(2xz-2xz)$
$= 0i - 0j + 0k = \langle 0,0,0\rangle \Rightarrow F$ is conservative

$r(0) = (0,0,1) \qquad r(2) = (4,1,1)$

Find F: $f(x,y,z) = \int 2xyz\,dx = x^2yz + g(y,z)$

$x^2z + g_y(y,z) = x^2z$

$g(y,z) = 0 + h(z)$

$f(x,y,z) = x^2z + h(z)$

$x^2 + h'(z) = x^2y - x^2$
$-x^2$

$h(z) = x^2yz - 2x^2$

$f(x,y,z) = \cancel{x^2z} + x^2yz - \cancel{2x^2} = x^2yz = f$

$f(0,0,1) = 0 \qquad f(4,1,1) = 16$

$16 - 0 = \boxed{16}$

For both paths integral must be the same

(19) $f = x^2y - z$ $\quad r_1 = \langle t, t, 0\rangle$ $0 \le t \le 1$ & $r_2 = \langle t, t^2, 0\rangle$ for $0 \le t \le 1$

$P = \langle 0,0,0\rangle$ $Q = \langle 1,1,0\rangle$ $\quad P = \langle 0,0,0\rangle$ $Q = \langle 1,1,0\rangle$

Beginning and End Points Are The Same For Both Curves

ask In Recitation

Find $F = \nabla f$

$= \langle 2xy, x^2, -1\rangle$

First curve:

$$\int_0^1 \langle 2xy, x^2, -1\rangle \cdot \langle dx, dy, dz\rangle$$

$r_1(t) = \langle t, t, 0\rangle$

$x(t) = t$ $y(t) = t$ $z(t) = 0$

$dx = dt$ $dy = dt$ $dz = 0$

$$\rightarrow \int_0^1 \langle 2t^2, t^2, -1\rangle \cdot \langle dt, dt, 0\rangle$$

$$= \int_0^1 3t^2\,dt = 3t^2\Big]_0^1 = \boxed{3}$$

Second Curve:

$r_2(t) = \langle t, t^2, 0\rangle$ $\quad x(t) = t$ $y(t) = t^2$ $z(t) = 0$

$dx = dt$ $dy = 2t\,dt$ $dz = 0$

$$\int_0^1 \langle 2t^3, t^2, -1\rangle \cdot \langle dt, 2t\,dt, 0\rangle$$

$$= \int_0^1 2t^3 + 2t^3\,dt = 4$$

$4 - 3 = \boxed{1}$