16.2

3. $F = \langle y^2, x^2 \rangle$, C $y = x^{-1}$ for $1 \le x \le 2$

a. $F(r(t)) = \langle t^{-2}, t^2 \rangle$

$r'(t) = (1, -t^{-2})$

b. $\langle t^{-2}, t^2 \rangle \cdot \langle 1, -t^{-2} \rangle = t^{-2} + 1$

$\int_1^2 (t^{-1} - 1)\,dt = \frac{1}{2} - \frac{1}{1} = \frac{-1}{2}$

9. $f(x,y) = \sqrt{1+9xy}$ $\quad y = x^3 \quad 0 \le x \le 1$

$f(t) = \sqrt{1+9t^4}$ $\quad x = t \quad y = t^3$

$ds = \sqrt{(1)^2 + (3t^2)^2}$

$\int_0^1 (1+9t^4)^{1/2} \cdot (1+9t^4)^{1/2}\,dt$ $\quad ds = \sqrt{1+9t^4}$

$\int_0^1 1 + 9t^4\,dt = t + \frac{9t^5}{5}\Big|_0^1 = \left(1 + \frac{9}{5}\right) - 0 = 2.8$

11. $f(x,y,z) = z^2 \quad r(t) = (2t, 3t, 4t) \quad 0 \le t \le 2$

$f(r(t)) = 16t^2$ $\quad \sqrt{(2)^2 + (3)^2 + (4)^2} = \sqrt{29}$

$\int_0^2 16t^2 \cdot \sqrt{29}\,dt$

$16\sqrt{29} \int_0^2 t^2\,dt = \frac{t^3}{3}\Big|_0^2 = 16\sqrt{29}\left(\frac{8}{3}\right) = \frac{128\sqrt{29}}{3} \approx 229.8$

13. $f(x,y,z) = xe^{z^2}$, $(0,0,1)$ $(0,2,0)$ $(1,1,1)$

$\langle 0, 2t, 1 \rangle$ $\sqrt{4+1} = \sqrt{5}$ $\quad \langle t, 2+t, t \rangle$ $\sqrt{1+1+1} = \sqrt{3}$ $\quad \langle 1, 1, 1+t \rangle$ $\sqrt{1+1+1} = \sqrt{3}$

$\int_0^1 0e^1 \cdot \sqrt{5}\,dt = 0$ $\quad \int_0^1 te^{t^2} \cdot \sqrt{3}\,dt = \frac{\sqrt{3}\,e^{t^2}}{2} = \frac{\sqrt{3}e - \sqrt{3}}{2}$ $\quad \int_0^1 1e^{(1+t)^2} \cdot \sqrt{3}\,dt$ $\quad = 0$

1.488

17. $\int_C 1\, ds$, $r(t) = (4t, -3t, 12t)$ $2 \le t \le 5$

$ds = \sqrt{(4^2) + (3^2) + (12^2)} = \sqrt{169} = 13\, dt$

$\int_2^5 13\, dt = 13\,(t)\Big|_2^5 = 13\,(5-2) = 13 \cdot 3 = 39$

This represents the distance between r(5) and r(2)

27. $\int_C y\,dx - x\,dy$, $y = x^2$ $0 \le x \le 2$

$x = t$ $y = t^2$

$x' = 1$ $y' = 2t$

$\int_0^2 t^2 \cdot 1 - t \cdot 2t\, ds$

$\int_0^2 -t^2\, dt = -\frac{t^3}{3} = -\frac{8}{3} - 0 = -\frac{8}{3}$

29. $\int (x-y)\,dx + (y-z)\,dy + z\,dz$ $(0,0,0)$ to $(1,4,4)$

$\langle t, 4t, 4t \rangle$ $dx = 1$ $dy = 4$ $dz = 4$

$\int_0^1 (-3t) \cdot 1 + (0) + 4t \cdot 4\, dt$

$\int_0^1 -3t + 16t\, dt = \frac{-3t^2}{2} + \frac{16t^2}{2}\Big|_0^1 = \left(\frac{-3}{2} + \frac{16}{2}\right) - 0 = \frac{13}{2}$

31. $\int_C \frac{-y\,dx + x\,dy}{x^2+y^2}$ $(1,0)$ to $(0,1)$

$\langle 1, t \rangle$ $dx = 0$ $dy = 1$

$\int_0^1 \frac{-t \cdot 0 + 1 \cdot 1}{1+t^2}\, dt$

$\int_0^1 \frac{1}{1+t^2}\, dt = \sin 1 - \sin 0 = \frac{\pi}{2}$

35. $F(x,y,z) = (e^z, e^{x-y}, e^y)$

$\langle 0, 0, t \rangle$

$\int_0^1 e^t + 2\, dt$

$e^t + 2t\Big|_0^1 = (e+2) - 1$

$e + 1$

$\langle 0, t, 1+t \rangle$

$\int_0^1 e^{1+t} + e^{-t} + e^t\, dt$

$2 - e - \frac{1}{e}$

$\langle -t, 1+t, 1+t \rangle$

$\int_0^1 e^{1+t} + e^{-2t+1} + e^{1+t}$

16.3

1. $f(x,y,z) = xy\sin(yz)$ and $F = \nabla f$. find $\int_C F \cdot dr$ $(0,0,0)$ to $(1,1,\pi)$

$F = y\sin(yz)\,i + x\sin(yz)\cdot z\,j + xy\sin(yz)\cdot y$

$$\int_{(0,0,0)}^{(1,1,\pi)} y\sin(yz)\,i + x\sin(yz)\cdot z\,j + xy\sin(yz)\cdot y = F(1,1,\pi) - F(0,0,0)$$

$1\sin(\pi) + 1\sin(\pi)\cdot 1 + 1\sin(\pi)\cdot 1 = 0 + 0 + 0 = 0$

3. $F(x,y) = \langle 3, 6y\rangle$, $f(x,y) = 3x + 3y^2$ $\quad r(t) = \langle t, 2t^{-1}\rangle$

$1 \le t \le 4$ $\quad r'(t) = \langle 1, -2t^{-2}\rangle$

$F\sqrt{1 + 4t^{-4}}$

$\langle 3, 6y\rangle = \langle \frac{d}{dx}(3x + 3y^2), \frac{d}{dy}(3x + 3y^2)\rangle$

$\langle 3, 6y\rangle = \langle 3, 6y\rangle$ $\quad \langle 3, 12t^{-1}\rangle$

$\int F \cdot dr = f(4) - f(1)$

$= 3(4) + 12(4^{-2}) - 3(1) + 12(1^{-2})$

$= 12.75 - 15$

$= -2.25$

5. $F(x,y,z) = ye^z\,i + xe^z\,j + xye^z\,k$ $\quad f(x,y,z) = xye^z$

$F = \nabla f$

$ye^z + xe^z + xye^z = \frac{d}{dx}xye^z + \frac{d}{dy}xye^z + \frac{d}{dz}xye^z$

$ye^z + xe^z + xye^z = ye^z + xe^z + xye^z$

$f(t) = t^5e^{t-1}$

$r(t) = (t^2, t^3, t-1)$ $\quad 1 \le t \le 2$

$F(r(t)) = t^3e^{t-1} + t^2e^{t-1} + t^5e^{t-1}$

$F(r(t)) = t^{10}e^{t-1}$

$\int F(r(t)) = f(2) - f(1) = 2^5e^1 - 1^5e^0 = 32e - 1$

9. $F = y^2 i + (2xy + e^z) j + ye^z k$

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ y^2 & (2xy+e^z) & ye^z \end{vmatrix} = (e^z - e^z)i + (0-0)j + (2y - 2y)k$$ conserv.

$F = \nabla f$

$y^2 i + (2xy + e^z) j + ye^z k = \frac{d}{dx} f + \frac{d}{dy} f + \frac{d}{dz} f$

$f = xy^2 + g(y,z)$

$2xy + e^z = 2xy + g'(y,z)$

$g'(y,z) = e^z$

$f = xy^2 + e^z + h(z)$

$ye^z = xy^2 + e^z + h(z)$

$ye^z = e^z + h'(z)$

$h(z) = ye^z$

$f = xy^2 + ye^z$

13. $F = \langle z\sec^2 x, z, y + \tan x \rangle$

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ z\sec^2 x & z & y+\tan x \end{vmatrix} = (1-1)i + (\sec^2 x - \sec^2 x)j + (0-0)k$$ cons.

$f = \int z\sec^2 x \, dx = z\tan x + g_y(y,z)$

$\int z = \int z\tan x + g_y(y,z)$

$zy = zy\tan x + g(y,z)$

$zy - zy\tan x = g(y,z)$

$g(y,z) = 0$

$z\tan x + xy - xy\tan x + h(z)$

$h'(z) = y + \tan x$

$h'(z) = zy$

$f = z\tan x + zy$

15. $F = \langle 2xy + 5, x^2 - 4z, -4y \rangle$

$f = x^2 y + 5x - 4zy$

$xy^2 + 5x + g'(y,z) = x^2 - 4z$

$g'(y,z) = 4zy$

17. $\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz$ $\quad 0 \le t \le 2$

$r(t) = \langle t^2, \sin(\frac{\pi t}{4}), e^{t^2-2t} \rangle$

$r(2) = \langle 4, \sin\frac{\pi}{2}, 1 \rangle = \langle 4, 1, 1 \rangle$

$r(0) = \langle 0, 0, 1 \rangle$

$x^2yz = x^2z + g(y)$

$g(y) = 0$

$x^2y = x^2yz + g(z)$

$x^2y = x^2y + g'(z)$

$g(z) = 0$

$x^2y \Big|_{(0,0,1)}^{(4,1,1)} = 4^2(1) - 0 = 16$

19. $f = x^2y - z$ $\quad r_1 = \langle t, t, 0 \rangle$ for $0 \le t \le 1$ $\quad r_2(t, t^2, 0)$ for $0 \le t \le 1$

$f(1,1,0) - f(0,0,0) = 1 - 0 = 1$

$f(1,1,0) - f(0,0,0) = 1 - 0 = 1$