Fayed Raza 11/13/20

16.2: 3, 9, 11, 13, ~~17~~, 27, 29, 31, 35

3. (a) $F(r(t)) = \langle t^2, t^{-2} \rangle$

$dr = \langle 1, -t^{-2} \rangle \, dt$

(b) $\langle t^{-2}, t^{-2} \rangle \cdot \langle 1, -t^{-2} \rangle$

$t^{-2} - t^{-4}$

$$\int_1^2 (t^{-2} - 1) \, dt$$

$\hookrightarrow -t^{-1} - t \Big|_1^2$

$-\frac{1}{2} - \cancel{2} + \cancel{1} + 1$

$(-1 - 1)$

$-\frac{1}{2}$

9. $$\int_0^1 \sqrt{1 + 9x^4} \sqrt{9x^4 + 1} \, dx$$

$\sqrt{9x^4+1}\ 3x^2$

$$\int_0^1 1 + 9x^4 \, dx$$

$x + \frac{9x^5}{5} \Big|_0^1$

$1 + \frac{9}{5} - 0$

$\frac{14}{5}$

1.

$(4t)^2$ $\qquad \sqrt{2^2+3^2+4^2}$ $\quad 4+9+16$

$$\int_0^2 16t^2\sqrt{29}\,dt \qquad \sqrt{29}$$

$$16\sqrt{29}\int_0^2 t^2\,dt$$

$$\left.\frac{t^3}{3}\right|_0^2$$

$$\frac{8}{3}-0$$

$$16\left(\frac{8}{3}\right)\sqrt{29}$$

$$\frac{128\sqrt{29}}{3}$$

13.

$(0,0,1)$ to $(0,2,0)$

$x=0$

$y=0+2t$

$z=1-t$

$(0,2,0)$ to $(1,1,1)$

$x=t$

$y=2-t$

$z=t$

$\sqrt{1^2+1^2+1^2}$

$$\int_0^1 0\,dt+\int_0^1 t\,e^{t^2}\sqrt{3}$$

$u=t^2$

$du=2t\,dt$

$$0+\frac{\sqrt{3}}{2}\int_0^1 e^u\,du$$

$$\frac{\sqrt{3}}{2}\quad e^1-1$$

$$\frac{\sqrt{3}}{2}(e-1)$$

17. $\sqrt{4^2+9+144} = 13$

$$\int_2^5 1(13)\,dt$$

$$13t\Big|_2^5$$

$$13(5)-26$$

$13 \times 5 = 65$

$65-26 = 39$

$y=x^2$  $\sqrt{y}=x$  $\frac{1}{2\sqrt{y}}=dx$

27. $\int_0^2 x^2\,(dx) - x(2x)\,dx$  $x=y^{\frac{1}{2}}$  $dx=\frac{1}{2\sqrt{y}}$

$\int_0^2 x^2-2x^2\,dx$  $\frac{x^2}{2x}$

$\frac{x^3}{3}-\frac{2x^3}{3}\Big|_0^2$

$\frac{8}{3}-\frac{16}{3}=-\frac{8}{3}$

$=\frac{13}{3}$

29. $x=0+t$
$y=4t$
$z=4t$

$\int_0^1 (-3t)\,dt + (4t)\,4\,dt$

$\int_0^1 16t-3t\,dt$

$\int_0^1 13t\,dt$  $\frac{13}{2}$

$\frac{13t^2}{2}\Big|_0^1$

31. $x = 1 - t$

$y = t$

$$\int_0^1 \frac{-t(-1) + (1-t)}{(1-t)^2 + t^2} \, dt$$

$(1-t)(1-t)$

$$\int_0^1 \frac{t \cancel{+1} - \cancel{t}}{1 - 2t + 2t^2} \, dt \qquad \frac{(1-t)(1-t)}{1 - 2t + t^2 + t^2}$$

$\frac{\pi}{2}$

$$\int_0^1 \frac{1}{1 - 2t + 2t^2} \, dt$$

$$\tan^{-1}(2x-1) \Big|_0^1 \qquad \int_0^1 \frac{1}{(\sqrt{2}x - \frac{1}{2})^2 + \frac{1}{2}} \, dx$$

$$\tan^{-1}(1) - \tan^{-1}(-1) = \frac{\pi}{2} - 0 = \frac{\pi}{2} \qquad \tan^{-1} \frac{1}{\sqrt{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{u^2 + 1} \, du \qquad u = \sqrt{2}t, \; du = \sqrt{2}\,dt$$

$\tan^{-1}(2x-1)$

33. $x = -t$

$y = t$

$z = t$

$$\int_0^1 -t^3 \sqrt{3} \, dt$$

$$-\frac{t^4}{4} \sqrt{3} \Big|_0^1$$

$$-\frac{\sqrt{3}}{4}$$

16.3: 1, 3, 5, 9, 13, 15, 17, 19 $\langle y\sin(y,z), x\sin(yz) + xyz\cos(z), \cos(yz) \rangle$

1. $x = t \quad y = t \quad z = \pi t$

$$\int_0^1 t y\sin(y,z) + t(\sin(y,z) + \cos(x,z)) + \pi t\, x(\cos yz)$$

$$\int_0^1 t^2\sin(t^2) + t(t\sin(\pi t^2) + \cos(\pi t^2)) + \pi t(\cos(\pi t^2))$$

$(1,0) - (1,0)$

$\sin(1) + 1(\sin(\pi) + \cos(\pi)) + \pi\cos(\pi)$

$-(1) + 0 + 0 \quad 0$

3. $F = \langle 3, 6y \rangle$ ✓

$\langle 3, 12t^{-1} \rangle \langle 1, -2t^{-3} \rangle$

$3 - 24t^{-3}$

$$\int_1^4 3 - 2y^{-3}$$

$$3t + 12t^{-2} \Big|_1^4$$

$3(4) + \frac{12}{16} - 3 - 12 \quad \frac{3}{4}$

$-3 + \frac{3}{4} \quad -\frac{12}{4} + \frac{3}{4} = \frac{9}{4}$

5 $F = \langle ye^z, xe^z, xye^z \rangle$

$F(r(t)) = \langle (t^3)e^{t-1}, t^2e^{t-1}, t^2(t^3)e^{t-1} \rangle$

$\langle t^3e^{t-1}, t^2e^{t-1}, t^5e^{t-1} \rangle \langle 2t, 3t^2, 1 \rangle$

$\int_1^2 2t^4e^{t-1} + 3t^4e^{t-1} + t^5e^{t-1}$

$\int_1^2 5t^4e^{t-1} + t^5e^{t-1}$

$\int_1^2 e^{t-1}(5t^4 + t^5)dt$

$32e - 1$

9. $\langle y^2, 2xy + e^z, ye^z \rangle$

$\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$ $2y$, $2y$ ✓

$\frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x}$ $0 = 0$ ✓

$\frac{\partial F_2}{\partial z} = \frac{\partial F_3}{\partial y}$ $e^z$, $e^z$ ✓

$\int y^2 dx$

$f = xy^2 + h(y, z)$

$f_y = 2xy + h_y(y, z)$

$\cancel{2xy} + e^z = \cancel{2xy} + h_y(y, z)$

$e^z = h_y(y, z)$

$h(y, z) = ye^z$

$f = xy^2 + e^z y$

13. $F = \langle z\sec^2 x, z, y + \tan x \rangle$ $f = z\tan x + zy$

$0 = 0$ ✓

$\sec^2 x = \sec^2 y$ ✓

$1 = 1$ ✓

$\int z\sec^2 x \, dx$

$f = z\tan x + h(y,z)$

$f_y = h_y(y,z)$

$z = h_y(y,z)$

$\Downarrow$

$zy$

15. $F = \langle 2xy + 5, x^2 - 4z, -4y \rangle$ $f = 2xy + 5 - 4yz$

$2x = 2x$ ✓

$0 = 0$ ✓

$-4 = -4$ ✓

$\int 2xy + 5 \, dx$

$f = x^2 y + 5x + h(x,y)$

$f_y = x^2 + h_y(x,y)$

$\cancel{x^2} - 4z = \cancel{x^2} + h_y(x,y)$

$-4z = h_y(x,y)$

$-4yz$

17. $\int_0^2 2(t^2)\sin(\pi t/4)e^{t^2-2t}(2t)$

$+(t^4)(\sin(\pi t/4))((2t-2)(\frac{\pi}{4})\cos(\frac{\pi t}{4})$

$+(t^4)(\sin(\pi t/4))(2t-2)(e^{t^2-2t})\,dt$

$16\int_0^2 t^3\sin(\pi t/4)\,e^{t^2-2t}$

$e^{t^2-2t}\Big|_0^2$

$e^{4-4}-e^0$

16

14. $\int x^2y-z$ $\langle 2xy, x^2, -1\rangle$

$2t^2+t^2$ $\frac{4}{4}$

$1=1\checkmark$ 1 $\langle 2t^2, t^2, -1\rangle\cdot\langle 1,1,0\rangle$ $\int_0^1 4t^3$

$\int_0^1 2t^2+t^2$

$\frac{2t^3}{3}+\frac{t^3}{3}\Big|_0^1$ $\frac{2}{3}+\frac{1}{3}=1$

$\langle 2t^3, t^2, 0\rangle$

$\langle 1, 2t, 0\rangle$

$2t^3+2t^3$