16.2 Homework

③ $F = \langle y^2, x^2 \rangle$, $y = x^{-1}$; $x = [1,2]$

a) $F(r(t)) = \langle t^{-2}, t^2 \rangle$ $\quad$ $r(t) = \langle t, t^{-1} \rangle$

$dr = \langle 1, -t^{-2} \rangle dt$ $\quad$ $r'(t) = \langle 1, -t^{-2} \rangle$

b) $F(r(t)) \cdot r'(t)\,dt = \langle t^{-2}, t^2 \rangle \cdot \langle 1, -t^{-2} \rangle = t^{-2} - 1$

$\int_1^2 (t^{-2} - 1)\,dt = -\frac{1}{2}$

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⑨ $f(x,y) = \sqrt{1+9xy}$; $y = x^3$; $0 \le x \le 1$

$\int_0^1 \sqrt{1+9x^4}\,dx = \frac{14}{5}$

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⑪ $f(x,y,z) = z^2$, $r(t) = \langle 2t, 3t, 4t \rangle$, $0 \le t \le 2$

$r'(t) = \langle 2, 3, 4 \rangle$, $|r'(t)| = \sqrt{29}$

$f(r(t)) = 16t^2$

$\int_0^2 (16t^2)(\sqrt{29})\,dt = \frac{128\sqrt{29}}{3}$

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⑬ $f(x,y,z) = xe^{z^2}$

$\int_C f(x,y,z) \cdot ds = \frac{\sqrt{3}}{2}(e-1)$

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⑰ $\int_C 1\,ds$ $\quad$ $r(t) = \langle 4t, -3t, 12t \rangle$, $2 \le t \le 5$

$r'(t) = \langle 4, -3, 12 \rangle \rightarrow |r'(t)| = 13$

$\int_2^5 13\,dt = 39$

㉗ $\int_C y\,dx - x\,dy$, $y = x^2$, $0 \le x \le 2$

$dy = 2x\,dx$

$$\int_0^2 x^2 dx - (x)(2x)\,dx = -\frac{8}{3}$$

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(29) $\int_C (x-y)\,dx + (y-z)\,dy + z\,dz$ from $(0,0,0)$ to $(1,4,4)$

$r(t) = \langle t, 4t, 4t \rangle$

$r'(t) = \langle 1,4,4 \rangle \Rightarrow |r'(t)| = \sqrt{33}$

$$\int_0^1 13t\,dt = \frac{13}{2}$$

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(31) $\int_C \frac{-y\,dx + x\,dy}{x^2+y^2}$ from $(1,0)$ to $(0,1)$

$r(t) = (1-t)\langle 1,0 \rangle + t\langle 0,1 \rangle$

$$\int_0^1 \left(\frac{t}{(1-t)^2+t^2} + \frac{1-t}{(1-t)^2+t^2} \right) dt = \frac{\pi}{2}$$

16.3 Homework

① $f(x,y,z) = xy\sin(yz)$

$\nabla f = \langle y\sin(yz),\ x(\sin(yz) + yz\cos(yz))\rangle$

$F = y\sin(yz)\,i + x(\sin(yz) + yz\cos(yz))\,j + xy^2\cos(yz)\,K$

$\int_C F \cdot dr = 0$

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③ $F(x,y) = \langle 3, 6y\rangle$, $f(x,y) = 3x + 3y^2$, $r(t) = \langle t, 2t^{-1}\rangle$, $1 \le t \le 4$

$\int_1^4 (3 - \frac{24}{t^3})\,dt = -\frac{9}{4}$

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⑤ $F(x,y,z) = ye^z i + xe^z j + xye^z K$, $f(x,y,z) = xye^z$

$r(t) = \langle t^2, t^3, t-1\rangle$ for $1 \le t \le 2$

$r'(t) = \langle 2t, 3t^2, 1\rangle$

$\int 6t^3e + 6t^3e + 6t^3e\,dt = \frac{135e}{2}$

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⑨ $F = y^2 i + (2xy + e^z)j + ye^z K$

$curl(F) = \langle 0,0,0\rangle$ f is conservative

$f = xy^2 + ye^z$

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⑬ $F = \langle z\sec^2(x), z, y + \tan(x)\rangle$

$curl(f) = \langle 0,0,0\rangle$ f is conservative

$f = z\tan x + yz$

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⑮ $F = \langle 2xy + 5, x^2 - 4z, -4y\rangle$

$f = x^2 - 4yz$

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17 $\int_0^2 F(r(t))\cdot r'(t)\,dt = 16$