16.2 HW - #3, 9, 11, 13, 17, 27, 29, 31, 35

3. $f(x,y,z) = x + yz$

$P = (0,0,0)$ to $(6,2,2)$

a) $f(r(t))$ and $ds = \|r'(t)\|dt$

for $r(t) = (6t, 2t, 2t)$ for $0 \le t \le 1$

b) $\int_c f(x,y,z)\,ds$

Based on $r(t)$...

$x(t) = 6t \rightarrow x'(t) = 6$

$y(t) = 2t \rightarrow y'(t) = 2$

$z(t) = 2t \rightarrow z'(t) = 2$

therefore,

$f(r(t)) = 6t + (2t)(2t) = 6t + 4t^2$

$\boxed{f(r(t)) = 6t + 4t^2}$

$ds = \|r'(t)\| = \sqrt{6^2 + 2^2 + 2^2} = \sqrt{44} = 2\sqrt{11}$

$\boxed{ds = 2\sqrt{11}}$

Line Integral:

$\int_c f(x,y,z)\,ds = \int_0^1 (6t + 4t^2)(2\sqrt{11})\,dt$

$= 2\sqrt{11} \int_0^1 6t + 4t^2\,dt = 2\sqrt{11}\left[3t^2 + \frac{4}{3}t^3\right]\Big|_0^1$

$= 2\sqrt{11}\left(3 + \frac{4}{3}\right) = \left(\frac{13}{3}\right)(2\sqrt{11}) = \boxed{\frac{26\sqrt{11}}{3}}$

9. $f(x,y) = \sqrt{1 + 9xy}$

$y = x^3$

$0 \le x \le 1$ $\Rightarrow$ Let $x = t$

therefore,

$y = t^3$

$0 \le t \le 1$

Given $x = t$ and $y = t^3$...

$x'(t) = 1$

$y'(t) = 3t^2$

$|r'(t)| = \sqrt{1^2 + (3t^2)^2} = \sqrt{1 + 9t^4}$

Line Integral: $\int_0^1 \left(\sqrt{1 + 9t^4}\right)\left(\sqrt{1 + 9t^4}\right)dt = \int_0^1 1 + 9t^4\,dt$

$= \left[t + \frac{9}{5}t^5\right]\Big|_0^1 = 1 + \frac{9}{5} = \boxed{\frac{14}{5}}$

11. $f(x,y,z) = z^2$

$r(t) = (2t, 3t, 4t)$

$0 \le t \le 2$

$r(t) = (2t, 3t, 4t)$, implies

- $x(t) = 2t \rightarrow x'(t) = 2$
- $y(t) = 3t \rightarrow y'(t) = 3$
- $z(t) = 4t \rightarrow z'(t) = 4$

$$\|r'(t)\| = \sqrt{2^2+3^2+4^2} = \sqrt{4+9+16} = \sqrt{29}$$

$$f(x(t), y(t), z(t)) = (4t)^2 = 16t^2$$

Line Integral: $\int_0^2 (16t^2)\sqrt{29}\,dt = \sqrt{29}\left[\frac{16}{3}t^3\right]\Big|_0^2 = \boxed{\frac{128\sqrt{29}}{3}}$

13. $f(x,y,z) = xe^{z^2}$

piecewise linear path

from $(0,0,1)$ to $(0,2,0)$ to $(1,1,1)$

$\int_c xe^{z^2} = \int_{c_1} xe^{z^2} + \int_{c_2} xe^{z^2}$ where $C_1$ is $(0,0,1)$ to $(0,2,0)$ and $C_2$ is $(0,2,0)$ to $(1,1,1)$

For $C1$:

Parametrize Equations:

$(0,0,1)(1-t) + (0,2,0) = (0,0,1-t) + (0,2,0) = (0,2,1-t)$

where $0 \le t \le 1$

implies...

$x(t) = 0 \rightarrow x'(t) = 0$

$y(t) = 2 \rightarrow y'(t) = 0$

$z(t) = 1-t \rightarrow z'(t) = -1$

$\|r'(t)\| = \sqrt{(-1)^2} = \sqrt{1} = 1$

$$\int_0^1 \left(0e^{(1-t)^2}\right) dt = [t]\Big|_0^1 = 1$$

For C2:

Parametrize Equations:

$(0,2,0)(1-t)+(1,1,1) = (0,\ 2-2t,0)+(1,1,1) = (1,3-2t,1),\ 0 \le t \le 1$

$r(t) = (1,3-2t,1)$

$x(t) = 1 \rightarrow x'(t) = 0$

$y(t) = 3-2t \rightarrow y'(t) = -2$ $\quad \|r'(t)\| = \sqrt{(-2)^2} = 4$

$z(t) = 1 \rightarrow z'(t) = 0$

$\int_0^1 (1)e^2(4)\,dt = 4\left[e^2 t\right]\Big|_0^1 = 4e^2$

$\int_C xe^{z^2} = 4e^2 + 1$

17. $\int_C 1\,ds$

$r(t) = (4t, -3t, 12t)$

$2 \le t \le 5$

$x(t) = 4t$

$y(t) = -3t$

$z(t) = 12$

$\|r'(t)\| = \sqrt{16+9+144} = \sqrt{169} = 13$

$\int_2^5 (1)(13)\,dt = 13t\Big|_2^5 = 13(5) - 13(2) = 65 - 26 = 39$

represents distance between $(4(2), -3(2), 12(2))$ and $(4(5), -3(5), 12(5))$

27. $\int_C y\,dx - x\,dy$

$y = x^2$

$0 \le x \le 2$

Let $x = t$ and $y = t^2$, $0 \le t \le 2$

$x(t) = t \rightarrow x'(t) = 1$

$y(t) = t^2 \rightarrow y'(t) = 2t$

$\|r'(t)\| = \sqrt{1^2 + (2t)^2} = \sqrt{1+4t^2}$

$\int_C y\,dx - x\,dy \Rightarrow (t^2)(1) - (t)(2t) = t^2 - 2t^2 = -t^2$

$\int_0^2 (-t^2)\sqrt{1+4t^2}\,dt$

29. $\int (x-y)\,dx + (y-z)\,dy + z\,dz$

from $(0,0,0)$ to $(1,4,4)$ $\qquad \|r'(t)\| = 0$

$(0,0,0)(1-t) + (1,4,4) = (1,4,4)$

$\int_0^1 (1-4)(0) + (4-4)(0) + (4)(0)(0)\,dt = [t]\Big|_0^1$

31. $\int_C \frac{-y\,dx + x\,dy}{x^2+y^2} \rightarrow \int \frac{-1 + (1-t)(0)}{(1-t)^2 + 1} = \int \frac{-1}{t^2-2t+2}$

$(1,0)$ to $(0,1)$ $\qquad (1-t)(1-t)$

$(1,0)(1-t) + (0,1) = (1-t, 0) + (0,1) = (1-t, 1)$

$r(t) = (1-t, 1)$

$x(t) = 1-t \rightarrow x'(t) = -1$

$y(t) = 1 \rightarrow y'(t) = 0$

$\Rightarrow |r'(t)| = \sqrt{(-1)^2} = 1$

$\int \frac{-1}{t^2-2t+2}\,dt = \left| \begin{array}{l} u = t^2 \\ du = 2t\,dt \end{array} \right|$

35. $P = (0,0,0)$ $\quad$ * blue path?

$Q = (-1,1,1)$

16.3 - # 1, 3, 5, 9, 13, 15, 17, 19

1. $f(x,y,z) = xy\sin(yz) = (1)(1)\sin(\pi) = 0$

$F = \nabla f$

$[t]|_0^1 = 1$

$(0,0,0)$ to $(1,1,\pi)$

r(t):

$(1,1,\pi)(1-t) = (1-t,\ 1-t,\ \pi-\pi t)$

$x(t) = 1-t$

$y(t) = 1-t$

$z(t) = \pi - \pi t$

3. $F(x,y) = \langle 3, 6y\rangle$

$\frac{dP}{dy} = \frac{dQ}{dx} \Rightarrow 0 = 0 \rightarrow$ conservative field so $F = \nabla f$

$f(x,y) = 3x + 3y^2$

$r(t) = \langle t, 2t^{-1}\rangle$

$1 \leq t \leq 4$

$f(t) = 3t + 3(2t^{-1}) = 3t + 6t^{-1} = 3t + \frac{1}{6t}$

$\|r'(t)\| = \sqrt{1 + (-4t^{-2})}$

$\int_1^4 \left(3t + \frac{1}{6t}\right)\left(\sqrt{1+(-4t^{-2})}\right) dt$

5. $F(x,y,z) = ye^z\,\hat{i} + xe^z\,\hat{j} + xye^z\,\hat{k}$

$f(x,y,z) = xye^z$

$r(t) = (t^2, t^3, t-1)$

$1 \leq t \leq 2$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ ye^z & xe^z & xye^z \end{vmatrix}$$

$= (xe^z - xe^z)\hat{i} - (ye^z - ye^z)\hat{j} + (e^z - e^z)\hat{k}$

$= 0\hat{i} + 0\hat{j} + 0\hat{k} \Rightarrow$ yes conservative

so $F = \nabla f$

9. $F = y^2 \hat{\imath} + (2xy + e^z)\hat{\jmath} + (ye^z)\hat{k}$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ y^2 & 2xy + e^z & ye^z \end{vmatrix} = (e^z - e^z)\hat{\imath} - (0 - 0)\hat{\jmath} + (2y - 2y)\hat{k}$$

$\Rightarrow$ conservative field

$f = \int y^2 \, dx = xy^2$

$f = \int (2xy + e^z) \, dy = xy^2 e^z$ $\quad f = xy^2 + xy^2 e^z + ye^z$

$f = \int (ye^z) \, dz = ye^z$

13. $F = \langle z\sec^2 x, z, y + \tan x \rangle$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ z\sec^2 x & z & y + \tan x \end{vmatrix} = (1 - 1)\hat{\imath} - (\sec^2 x - \sec^2 x)\hat{\jmath} + (0 - 0)\hat{k}$$

$\Rightarrow$ conservative field

$f = \int z\sec^2 x \, dx = z\tan x$

$f = \int z \, dy = zy$ $\quad f = zy + z\tan x$

$f = \int y + \tan x \, dz = yz + z\tan x$

15. $F = \langle 2xy+5,\ x^2-4z,\ -4y \rangle$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 2xy+5 & x^2-4z & -4y \end{vmatrix} = (-4+4)\hat{\imath} - (0-0)\hat{\jmath} + (2x-2x)\hat{k}$$

$\Rightarrow$ conservative field

$f = \int (2xy+5)\,dx \rightarrow f = x^2y + 5x$

$f = \int (x^2-4z)\,dy \rightarrow f = yx^2 - 4zy$

$f = \int (-4y)\,dz \rightarrow f = -4yz$

$\Big]\ f = yx^2 + 5x - 4yz$

17. $\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz$

$r(t) = (t^2, \sin(\frac{\pi t}{4}), e^{t^2-2t})\quad 0 \le t \le 2$

$$\int_0^2 2(t^2)(\sin\tfrac{\pi t}{4})(e^{t^2-2t})(2t) + (t^4)(e^{t^2-2t})(\cos(\tfrac{\pi t}{4})\tfrac{\pi}{4})$$
$$+ (t^4)(\sin(\tfrac{\pi t}{4}))(e^{t^2-2t})(2t-2)$$

$= 8\sin(\frac{\pi}{2})(4) + 16\cos(\frac{\pi}{2})\frac{\pi}{4} + 16\sin(\frac{\pi}{2})(2)$

19. $f = x^2y - z$

$r_1\ \langle t, t, 0\rangle$ for $0 \le t \le 1 \rightarrow f = t^3 \rightarrow 1-0=1$

$r_2 = \langle t, t^2, 0\rangle$ for $0 \le t \le 1 \rightarrow f = t^4 \rightarrow f(1)-f(0) = 1-0=1$