16.2 HW

3. $F = \langle y^2, x^2 \rangle$ $\quad y = x^{-1}$ $\quad 1 \le x \le 2$

a. $F(r(t))$ $\quad dr = r'(t)\,dt$ $\quad r(t) = \langle t, t^{-1} \rangle$

$x = t,\ y = t^{-1}$

$F(r(t)) = \langle t^{-2}, t^2 \rangle$

$dr = r'(t)\,dt$

$dr = \langle 1, -1/t^2 \rangle\,dt$

b. $\int_C F \cdot dr$

$= \int_1^2 \left(\frac{1}{t^2} - 1\right) dt$ $\quad x = t$

$= -\frac{1}{2}$

9. $f(x,y) = \sqrt{1+9xy}$ $\quad y = x^3$ $\quad 0 \le x \le 1$

$\int_C f\,ds$ $\quad \int_a^b f(r(t))\,\|r'(t)\|\,dt$

$\sqrt{1+9xy}$ $\quad x = t$ $\quad y = t^3$

$r(t) = \langle t, t^3 \rangle$ $\quad 0 \le t \le 1$

$f(r(t)) = \sqrt{1+9(t)(t^3)} = \sqrt{1+9t^4}$ $\quad r'(t) = \langle 1, 3t^2 \rangle$

$\sqrt{1^2 + (3t^2)^2} = \sqrt{1+9t^4}$

$\int_0^1 \sqrt{1+9t^4} \cdot \sqrt{1+9t^4}\,dt$

$= \int_0^1 1+9t^4\,dt = \frac{14}{5}$

11. $f(x,y,z) = z^2$ $\quad r(t) = \langle 2t, 3t, 4t \rangle$ $\quad 0 \le t \le 2$

$x = 2t,\ y = 3t,\ z = 4t$

$f(r(t)) = 16t^2$

$r'(t) = \langle 2, 3, 4 \rangle$

$\|r'(t)\| = \sqrt{29}$

$\int_0^2 16t^2\sqrt{29}\,dt = \left.\frac{16t^3 \cdot \sqrt{29}}{3}\right|_0^2$

P Q R

13. $f(x,y,z) = xe^{z^2}$ $(0,0,1)$ to to $(0,2,0)$ to $(1,1,1)$

2 lines
PQ and QR
then find integrals and add

$PQ = (1-t)\langle 0,0,1\rangle + t\langle 0,2,0\rangle$
$r(t) = \langle 0, 2t, 1-t\rangle$
$r'(t) = \langle 0, 2, -1\rangle$
$\|r'(t)\| = \sqrt{4+1} = \sqrt{5}$
$f(r(t)) = 0 \cdot e^{(1-t)^2} = 0$
$\int_0^1 0 \cdot \sqrt{5}\, dt = 0$

QR
$r(t) = (1-t)\langle 0,2,0\rangle + t\langle 1,1,1\rangle$
$= \langle t, 2-t, t\rangle$
$r'(t) = \langle 1, -1, 1\rangle$
$\|r'(t)\| = \sqrt{1+1+1} = \sqrt{3}$
$f(r(t)) = t \cdot e^{t^2}$
$\int_0^1 te^{t^2}\sqrt{3}\, dt + 0$

17. $r(t) = \langle 4t, -3t, 12t\rangle$
$r'(t) = \langle 4, -3, 12\rangle$
$\|r'(t)\| = \sqrt{16+9+144} = 13$
$\int_C 1\, ds$ $ds = 13\, dt$

$f(r(t)) = 1$
$\int_2^5 1 \cdot 13\, dt = 13t\Big|_2^5 = 13(5-2) = 39$

Represents length of line segment from point $r(2)$ to $r(5)$

27. $\int_C y\,dx - x\,dy \qquad y = x^2 \qquad 0 \le x \le 2$

$r(t) = \langle t, t^2 \rangle$

$r'(t) = \langle 1, 2t \rangle\, dt$

$\|r'(t)\| = \sqrt{1+4t^2}$

$\int (y, -x) \cdot (dx, dy)$

$= \int_0^2 (t^2, -t) \cdot (1, 2t)\, dt = \int_0^2 t^2 - 2t^2\, dt = \int_0^2 -t^2\, dt$

$= -\frac{t^3}{3}\Big|_0^2 = -\frac{(2-0)^3}{3} = \frac{-8}{3}$

29. $\int_C (x-y)\,dx + (y-z)\,dy + z\,dz \qquad (0,0,0) \text{ to } (1,4,4)$

$r(t) = (1-t)\langle 0,0,0 \rangle + t\langle 1,4,4 \rangle$

$= \langle t, 4t, 4t \rangle \qquad x = t \quad z = 4t \quad y = 4t$

$\int_C (x-y, y-z, z) \cdot (dx, dy, dz) \qquad r'(t) = \langle 1, 4, 4 \rangle$

$\int_0^1 (t-4t, 0, 4t) \cdot (1, 4, 4)$

$= \int_0^1 t - 4t + 16t = \int_0^1 13t = \frac{13t^2}{2}\Big|_0^1 = \frac{13(1-0)}{2} = \frac{13}{2}$

31. $\int_C \frac{-y\,dx + x\,dy}{x^2+y^2} \qquad (1,0) \text{ to } (0,1)$

$r(t) = (1-t)\langle 1,0 \rangle + t\langle 0,1 \rangle$

$= \langle 1-t, 0 \rangle + \langle 0, t \rangle$

$= \langle 1-t, t \rangle$

$r'(t) = \langle -1, 1 \rangle$

$\int_0^1 \frac{(-t)(-dt) + (1-t)(dt)}{(1-t)^2 + (t)^2} = \frac{t\,dt + dt - t\,dt}{1-2t+t^2+t^2} = \int_0^1 \frac{dt}{2t^2-2t+1}$

35. $(0,0,0) \to (0,0,1)$ + Line Integral
$(0,0,1) \to (0,1,1)$ +
$(0,1,1) \to (-1,1,1)$

First: $(1-t)\langle 0,0,0\rangle + t\langle 0,0,1\rangle = \langle 0,0,t\rangle$

$r'(t) = \langle 0,0,1\rangle$ $\quad s = r'(t)$

$\|r'(t)\| = 1$

$f(r(t)) \cdot \langle 0,0,1\rangle = 1$

$\int_0^1 1\,dt = 1$

Second: $(1-t)\langle 0,0,1\rangle + t\langle 0,1,1\rangle = \langle 0,t,1\rangle$

$r'(t) = \langle 0,1,0\rangle$

$f(r(t)) \cdot \langle 0,1,0\rangle = e^{-t}$

$\int_0^1 e^{-t}\,dt = -1+e^{-1}$

Third: $(1-t)\langle 0,1,1\rangle + t\langle -1,1,1\rangle = \langle -t,1,1\rangle$

$r'(t) = \langle -1,0,0\rangle$

$f(r(t)) \cdot \langle -1,0,0\rangle = -e$

$\int_0^1 -e\,dt = -e$

$1 + 1 - e^{-1} - e = 2 - e^{-1} - e$

16.3 Homework

1. $f(x,y,z) = xy\sin(yz)$

$F = \nabla f$

$$\int_C F \cdot dr = f(1,1,\pi) - f(0,0,1)$$

$$= 0$$

3. $F(x,y) = \langle 3, 6y \rangle$ $\quad f(x,y) = 3x + 3y^2$ $\quad r(t) = \langle t, 2t^{-1} \rangle$

$1 \leq t \leq 4$

$\nabla f = \langle 3, 6y \rangle$ ✓

$F(r(t)) = \langle 3, 12t^{-1} \rangle$ $\quad r'(t) = \langle 1, -2/t^2 \rangle \, dt$

$$\int_1^4 \langle 3, 12t^{-1} \rangle \cdot \langle 1, -2/t^2 \rangle \, dt$$

$$= \int_1^4 \left(3 - \frac{24}{t^3}\right) dt = 3t + \frac{12}{t^2} \Big|_1^4$$

$$= -\frac{1}{4}$$

5. $F(x,y,z) = \langle ye^z, xe^z, xye^z \rangle$

$f(x,y,z) = xye^z$

$\nabla f = \langle ye^z, xe^z, xye^z \rangle$ ✓

$r(t) = \langle t^2, t^3, t-1 \rangle \quad 1 \leq t \leq 2$

$r'(t) = \langle 2t, 3t^2, 1 \rangle$

$F(r(t)) = \langle t^3 e^{t-1}, t^2 e^{t-1}, t^5 e^{t-1} \rangle$

$$\int_1^2 F(r(t)) \cdot r'(t) \cdot dt$$

$F(r(t)) \cdot r'(t) = 2t^4 e^{t-1} + 3t^4 e^{t-1} + t^5 e^{t-1}$

$= 5t^4 e^{t-1} + t^5 e^{t-1}$

$$\int_1^2 5t^4 e^{t-1} + t^5 e^{t-1} \cdot dt$$

9. $F = \langle y^2, 2xy + e^z, ye^z \rangle$

$F_1 \quad F_2 \quad F_3$

$\frac{\partial F_1}{\partial y} = 2y$ ✓ $\frac{\partial F_2}{\partial x} = 2y$ ✓ It is conservative

$\frac{\partial f}{\partial x} = y^2 \qquad \frac{\partial f}{\partial y} = 2xy + e^z \qquad \frac{\partial f}{\partial z} = ye^z$

$f(x,y,z) = \int y^2\,dx = xy^2 + g(y,z)$

$\frac{\partial f}{\partial y} = 2xy + \frac{\partial g}{\partial y} = 2xy + e^z$

$\frac{\partial g}{\partial y} = e^z$

$F(y,z) = \int e^z\,dy = ye^z + h(z)$

$f(x,y,z) = xy^2 + ye^z + g(z)$

$\frac{\partial f}{\partial z} = ye^z + \frac{\partial g}{\partial z} = ye^z \qquad \frac{\partial g}{\partial z} = 0$

so $g(z)$ is some constant

$f(x,y,z) = xy^2 + ye^z$

13. $F = \langle z\sec^2 x, z, y + \tan x \rangle$

$F_1 \quad F_2 \quad F_3$

$\frac{\partial F_1}{\partial y} = 0 \qquad \frac{\partial F_2}{\partial x} = 0$ ✓ It is conservative

$\frac{\partial f}{\partial x} = z\sec^2 x \qquad \frac{\partial f}{\partial y} = z \qquad \frac{\partial f}{\partial z} = y + \tan x$

$f(x,y,z) = \int z\sec^2 x\,dx = z\tan x + g(y,z)$

$f(x,y,z) = z\tan x + g(y,z)$

$\frac{\partial f}{\partial y} = 0 + \frac{\partial g}{\partial y} = z$

$\frac{\partial g}{\partial y} = z$

$g(y,z) = \int z\,dy = yz + h(z)$

$f(x,y,z) = z\tan x + yz + h(z)$

$\frac{\partial f}{\partial z} = \tan x + y + \frac{\partial h}{\partial z} = y + \tan x$

$\frac{\partial h}{\partial z} = 0$ so $h(z)$ is some constant

$f(x,y,z) = z\tan x + yz$

15. $F = \langle 2xy + 5,\ x^2 - 4z,\ -4y \rangle$

$\quad\ \ F_1 \qquad\qquad F_2 \qquad\quad F_3$

$\frac{\partial F_1}{\partial y} = 2x \qquad \frac{\partial F_2}{\partial x} = 2x$ ✓ It is conservative

$\frac{\partial f}{\partial x} = 2xy + 5 \qquad \frac{\partial f}{\partial y} = x^2 - 4z \qquad \frac{\partial f}{\partial z} = -4y$

$f(x,y,z) = \int 2xy + 5\,dx = x^2y + 5x + g(y,z)$

$\frac{\partial f}{\partial y} = x^2 + \frac{\partial g}{\partial y} = x^2 - 4z \implies \frac{\partial g}{\partial y} = -4z$

$g(y,z) = \int -4z\,dy = -4yz + h(z)$

$f(x,y,z) = x^2y + 5x - 4yz + h(z)$

$\frac{\partial f}{\partial z} = -4y + \frac{\partial h}{\partial z} = -4y \qquad \frac{\partial h}{\partial z} = 0$ so $h(z)$ is some constant

$f(x,y,t) = x^2y + 5x - 4yz$

17. $\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz$

$r(t) = (t^2, \sin(\pi t/4), e^{t^2-2t}) \quad 0 \le t \le 2$

$F = \langle 2xyz, x^2z, x^2y \rangle$

$F_1 \quad F_2$

$\frac{\partial F_1}{\partial y} = 2xz \qquad \frac{\partial F_2}{\partial x} = 2xz$ ✓ Conservative

$\frac{\partial f}{\partial x} = 2xyz \qquad \frac{\partial f}{\partial y} = x^2z \qquad \frac{\partial f}{\partial z} = x^2y$

$f(x,y,z) = \int 2xyz\,dx = x^2yz + g(y,z)$

$\frac{\partial f}{\partial y} = x^2z + \frac{\partial g}{\partial y} = x^2z \qquad \frac{\partial g}{\partial y} = 0$

$g(y,z) = \int 0\,dy = 0 + h(z)$

$f(x,y,z) = x^2yz + h(z)$

$\frac{\partial f}{\partial z} = x^2y + \frac{\partial h}{\partial z} = x^2y \qquad \frac{\partial h}{\partial z} = 0$ again some constant

$f(x,y,z) = x^2yz$

$f(r(b)) - f(r(a)) = (16) - (0) = \boxed{16}$

$r(2) = \langle 4, 1, 1 \rangle$

$r(0) = \langle 0, 0, 1 \rangle$

19. $F = \langle 2xy, x^2, -1 \rangle$

C_1 $f(1,1,0) - f(0,0,0)$

$= 1$

C_2 $f(1,1,0) - f(0,0,0)$

$= 1$ ✓