

16.2

3. Let  $F = (y^2, x^2)$  and let  $C$  be the curve  $y = x^{-1}$  for  $1 \leq x \leq 2$  oriented from left to right

a)  $x = t$   $y = t^{-1}$   $y = \ln F = (y^2, x^2)$   $F(c(t)) = (t^{-2}, t^2) = (t^{-2}, t^2)$   
 $c(t) = \frac{d}{dt}(t, t^{-1}) = (1, -t^{-2})$   $ds = (1, -t^{-2}) dt$   
 b) dot product:  $F(c(t)) \cdot c'(t) = (t^{-2}, t^2) \cdot (1, -t^{-2}) = \boxed{t^{-2} - 1}$

9.  $f(x, y) = \sqrt{1+9xy}$ ,  $y = x^2$  for  $0 \leq x \leq 1$   $\int_0^1 \sqrt{1+9x^4} dx = 14/5$

11.  $f(x, y, z) = z^2$   $c(t) = (2t, 3t, 4t)$ ,  $0 \leq t \leq 2$   $r'(t) = (2, 3, 4)$   
 $|r'(t)| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$   $f(r(t)) = 16t^2$   $\int_0^2 (16t^2)(\sqrt{29}) dt = \frac{128\sqrt{29}}{3}$

13.  $f(x, y, z) = xe^{z^2}$   $(0, 0, 1) \rightarrow (0, 2, 0) \rightarrow (1, 1, 1)$   $\int_C f(x, y, z) ds = \frac{\sqrt{2}}{2} (e - 1)$

17.  $\int_C ds$   $r(t) = (4t, -3t, 12t)$   $2 \leq t \leq 5$   $r'(t) = (4, -3, 12) \rightarrow |r'(t)| = \sqrt{16+9+144} = 15$   
 $\int_2^5 (1)(15) dt = 39$

27.  $\int_C y dx - x dy$ ,  $y = x^2$ ,  $0 \leq x \leq 2$   $dy = 2x dx$   $\int_0^2 x^2 dx - (x)(2x) dx = -8/3$

29.  $\int_C (x-4) dx + (y-2) dy + z dz$  from  $(0, 0, 0)$  to  $(1, 4, 4)$   
 $r(t) = (t, 4t, 4t)$   $r'(t) = (1, 4, 4) \rightarrow |r'(t)| = \sqrt{33}$   $F(r(t)) = (t-4, 4t-2, 4t)$   
 $\int_0^1 (13) dt = 13/2$

31.  $\int_C \frac{-y dx + x dy}{x^2 + y^2}$   $(1, 0)$  to  $(0, 1)$   $= \pi/2$

35.  $\boxed{2 - \frac{1}{e} + e}$

16.3

1.  $f(x, y, z) = xy \sin(yz)$   $\nabla f = \langle y \sin(yz), x(\sin(yz) + yz \cos(yz)), xy^2 \cos(yz) \rangle$   
 $P = y \sin(yz) \mathbf{i} + x(\sin(yz) + yz \cos(yz)) \mathbf{j} + xy^2 \cos(yz) \mathbf{k}$   
 Start:  $f(0,0,0) = 0$  end:  $f(1,1,1) = 0$   $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$

2.  $\mathbf{F}(x, y) = \langle 2, 6y \rangle$ ,  $f(x, y) = 3x + 3y^2$ ,  $\mathbf{r}(t) = \langle t, 2t^2 \rangle$ ,  $1 \leq t \leq 4$   $\mathbf{r}'(t) = \langle 1, 4t \rangle$   
 $= -9/4$

5.  $\mathbf{F}(x, y, z) = ye^z \mathbf{i} + xe^z \mathbf{j} + xye^z \mathbf{k}$ ,  $f(x, y, z) = xye^z$ ,  $f(x, y, 0) = xye^0$   
 $\int_0^2 (6t^2 e + 6t^2 e + 6t^2 e) dt = \int_0^2 18t^2 dt = \frac{135e}{2}$

9.  $\mathbf{F} = ye^z \mathbf{i} + (2xy + e^z) \mathbf{j} + ye^z \mathbf{k}$   $\text{curl}(\mathbf{F}) = \langle 0, 0, 0 \rangle$   $\mathbf{F}$  is conservative  
 $f = xy^2 + ye^z$

13.  $\mathbf{F} = \langle 2z \sec^2 x, z, y + \tan x \rangle$   $f = z \tan x + yz$

15.  $\mathbf{F} = \langle 2xy + 5, y^2 - 4z, -4y \rangle$   $f_x = 2xy + 5$ ,  $f_y = x^2 - 4z$ ,  $f_z = -4y$   
 $f = x^2 - 4yz$

17.  $\int_C 2xyz dx + x^2 z dy + x^2 y dz$ ,  $\mathbf{r}(t) = \langle t^2, \sin(\pi t/4), e^{1-t} \rangle$   
 $\mathbf{r}'(t) = \langle 2t, \frac{\pi \cos(\pi t/4)}{4}, (2t-2)e^{1-t} \rangle$   
 $\int_0^2 \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt = \boxed{16}$