16.2/16.3 Homework

16.2

3. a. $x = t$ $\quad 1 \le t \le 2$

$y = t^{-1}$

$$\boxed{F(r(t)) = \langle t^{-2}, t^{2} \rangle}$$

$dr = r'(t)dt$

$\frac{dr}{dt} = r'(t) \qquad r(t) = (t, t^{-1})$

$$\boxed{ds = (1, -t^{-2})\,dt}$$

b. $= \langle t^{-2}, t^{2} \rangle \cdot \langle 1, -t^{-2} \rangle$

$$\boxed{= t^{-2} - 1}$$

9. $x = t \quad y = t^{3} \qquad 0 < t \le 1$

$$\int_C f\,ds = \int_0^1 \sqrt{1+9t^4}\,\sqrt{1+9t^4}\,dt$$

$$= \int_0^1 1 + 9t^4\,dt$$

$$= t + \frac{9}{5}t^5 \Big|_0^1 = \boxed{2.8}$$

11. $f(x,y,z) = z^2 \quad r(t) = (2t, 3t, 4t)$

$0 \leq t \leq 2$

$= \int 16t^2 \sqrt{4+9+16}\, dt$

$= 16\sqrt{29} \int_0^2 t^2 \, dt$

$= 16\sqrt{29}\,(8)/3 = \boxed{\frac{128}{3}\sqrt{29}}$

13. $f(x,y,z) = xe^{z^2}$

Integral 1

$x(t) = (1-t)0 + 0t = 0$

$y(t) = (1-t)0 + 2t = 2t$

$z(t) = (1-t)1 + 0t = 1-t$

$0 \leq t \leq 1$

$\sqrt{0+4+1} = \sqrt{5}$

$= \int_0^1 0 \, dt = 0$

Integral 2

$x(t) = (1-t)0 + 1t = t$

$y(t) = (1-t)2 + 1t = 2-t$

$z(t) = (1-t)0 + 1t = t$

$0 \leq t \leq 1$

$\sqrt{1+1+1} = \sqrt{3}$

$\sqrt{3}\int_0^1 te^{t^2} \, dt$

$= \sqrt{3} \left. \frac{e^{t^2}}{2} \right|_0^1$

$= \sqrt{3}\left(e/2 - \frac{1}{2}\right) = \boxed{\frac{\sqrt{3}(e-1)}{2}}$

17. $\sqrt{16+9+144} = 13$

$$\int_2^5 13 = 13t\Big|_2^5 = \boxed{39}$$

Distance from (8, −6, 24) to (20, −15, 60)

27. $\int_C y\,dx - x\,dy$ $\quad y = x^2$ $\quad 0 \le x \le 2$

$x = t$ $\quad 0 \le t \le 2$

$y = t^2$

$$\int_0^2 t^2 - 2t^2\,dt = \int_0^2 -t^2\,dt$$

$$= \frac{-t^3}{3}\Big|_0^2 = \boxed{\frac{-8}{3}}$$

29. $\int_C (x-4)\,dx + (y-z)\,dy + z\,dz$

$x(t) = (1-t)0 + t = t$

$y(t) = (1-t)0 + 4t = 4t$

$z(t) = (1-t)0 + 4t = 4t$

$0 \le t \le 1$

$$\int_0^1 -3t + 16t\,dt = \int_0^1 13t$$

$$= \frac{13t^2}{2}\Big|_0^1 = \boxed{\frac{13}{2}}$$

31. $\int_C \frac{-y\,dx + x\,dy}{x^2+y^2}$

$x(t) = (1-t)1 = 1-t$

$y(t) = (1-t)0 + t = t$

$0 \le t \le 1$

$$= \int_0^1 \frac{t + 1 - t}{1 - 2t + 2t^2} = \int_0^1 \frac{1}{1-2t+2t^2}$$

used maple because I forgot what that integral equals (arctan)

$\boxed{= \frac{\pi}{2}}$

35. Integral 1

$\int F(r(t)) \cdot r'(t) = 1$

$x=0$
$y=0 \quad 0 \le t \le 1$
$z=t$

Integral 2

$x=0$
$y=t$
$z=1$

$\int F(r(t)) \cdot r'(t) =$

$\int_0^1 e^{-t} = -e^{-t}\Big|_0^1 = 1-\frac{1}{e}$

Integral 3

$x=-t$
$y=1$
$z=1$

$\int F(r(t)) \cdot r'(t) =$

$\int_0^1 -e = -et\Big|_0^1 = -e$

Total: $\boxed{2-e-\frac{1}{e}}$

16.3

1. $= f(1,1,2) - f(0,0,0)$
$= 0 - 0 = \boxed{0}$

3. $f_x = 3 \quad \nabla f = \langle 3, 6y \rangle$
$f_y = 6y$

$r(4) = \langle 4, 1/2 \rangle$
$r(1) = \langle 1, 2 \rangle$

$f(r(4)) - f(r(1))$
$= 12 + \frac{3}{4} - 15$

$-3 + \frac{3}{4} = \boxed{\frac{-9}{4}}$

5. $f_x = ye^z$
$f_y = xe^z$
$f_z = xye^z$

$\nabla f = \langle ye^z, xe^z, xye^z \rangle$

$r(2) = \langle 4, 8, 1 \rangle$ $\quad r(1) = \langle 1, 1, 0 \rangle$

$f(r(2)) - f(r(1)) = \boxed{32e - 1}$

9.

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & 2xy + e^z & ye^z \end{vmatrix} = i(e^z - e^z) - j(0 - 0) + k(2y - 2y)$$

$= 0$, field is conservative

$F = \langle y^2, 2xy + e^z, ye^z \rangle$

$f(x,y,z) = xy^2 + h(y,z)$

$2xy + h_y(y,z) = 2xy + e^z$

$h_y = e^z$

$h(y,z) = ye^z + g(z)$

$f(x,y,z) = xy^2 + ye^z + g(z)$

$ye^z + g'(z) = ye^z \quad g'(z) = 0$

$g(z) = 0$

$\boxed{f(x,y,z) = xy^2 + ye^z}$

13. $\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z\sec^2 x & z & y+\tan x \end{vmatrix}$

$= i(1-1) - j(\sec^2 x - \sec^2 x) + k(0-0)$

$= 0$, field is conservative

$f(x,y,z) = z\tan x + h(y,z)$

$0 + h_y(y,z) = z$

$h_y(y,z) = z$

$h(y,z) = yz + g(z)$

$f(x,y,z) = z\tan x + yz + g(z)$

$\tan x + y + g'(z) = y + \tan x$

$g'(z) = 0 \quad g(z) = 0$

$\boxed{f(x,y,z) = z\tan x + yz}$

15. $\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy+5 & x^2-4z & -4y \end{vmatrix}$

$= i(-4-(-4)) - j(0-0) + k(2x-2x)$

$=0$, field is conservative

$f(x,y,z) = x^2y + 5x + h(y,z)$

$x^2 + h_y(y,z) = x^2 - 4z$

$h_y(y,z) = -4z \quad h(y,z) = -4yz + g(z)$

$f(x,y,z) = x^2y + 5x - 4yz + g(z)$

$-4y + g'(z) = -4y$

$g(z) = 0, \; g(z) = 0$

$\boxed{f(x,y,z) = x^2y + 5x - 4yz}$

17. $f(x,y,z) = x^2yz + h(y,z)$

$x^2z + h_y(y,z) = x^2z$

$h_y(y,z) = 0, \; h(y,z) = 0$

$g(z) = 0$

$f(x,y,z) = x^2yz$

$r(2) = \langle 4, 1, 1 \rangle$

$r(0) = \langle 0, 0, 1 \rangle$

$f(r(2)) - f(r(0))$

$= 16 - 0 = \boxed{16}$

19. $r_1(0) = \langle 0,0,0 \rangle$ $r_2(0) = \langle 0,0,0 \rangle$

$r_1(1) = \langle 1,1,0 \rangle$ $r_2(1) = \langle 1,1,0 \rangle$

Corresponding vectors are identical,
So path integrals are also equal