17) Ans. Plot(C)

16.2

3. $F = \langle y^2, x^2 \rangle$ and let C be the curve $y = \frac{1}{x}$ for $1 \le x \le 2$ oriented from left to right

(a) $x(t) = t \quad x'(t) = 1 \quad 1 \le t \le 2$

$y(t) = \frac{1}{t} \quad y'(t) = -\frac{1}{t^2}$

$$r'(t) = \left\langle 1, -\frac{1}{t^2} \right\rangle$$

(b) $F = \langle y^2, x^2 \rangle = \left\langle \frac{1}{t^2}, t^2 \right\rangle$

$= F \cdot d(r)$

$$\left\langle \frac{1}{t^2}, t^2 \right\rangle \cdot \left\langle 1, -\frac{1}{t^2} \right\rangle$$

$$\frac{1}{t^2} - 1$$

9) $f(x,y) = \sqrt{1+9xy}$ $y = x^3$ for $0 \le x \le 1$

$ds = \sqrt{1+(3t^2)^2}$

$= \sqrt{1+9t^4}$

$f(x,y) = \sqrt{1+9t^4}$

$f(x,y) \cdot ds$

$= \int_0^1 1+9t^4$

$= \left[t + \frac{9t^5}{5} \right]_0^1$

$= 1 + \frac{9}{5}$

$= \frac{14}{5}$

11) $f(x,y,z) = z^2$

$r(t) = (2t, 3t, 4t)$ for $0 \leq t \leq 2$

$r'(t) = (2, 3, 4)$

$|r'(t)| = \sqrt{4+9+16}$

$ds = \sqrt{29}$

$f(x,y,z) = 16t^2$

$$\left[16\sqrt{29}\ \frac{t^3}{3}\right]_0^2$$

$$\sqrt{29} \times \frac{4}{3} \times 16$$

13) $f(x,y,z) = xe^{z^2}$

$AB = P + t(Q-P)$

$\langle 0,0,1\rangle + t\langle 0,2,-1\rangle$

$u = \langle 0, 2t, 1-t\rangle$

$A \to B = 0$

$A \to C$ –

$AC = \langle 0,0,1\rangle + t\langle 1,1,0\rangle$

$= \langle t, t, 1\rangle$

$f = te$

$\int_0^1 te$

$\frac{et^2}{2}$

$$= \frac{e}{2}$$

17) $r'(t) = \langle 4, -3, 12 \rangle$

$$|r'(t)| = 16 + 9 + 144$$

$$= 13$$

$$[13t]_2^5$$

$$13 \times 3$$

$$= 39$$

27) $y = x^2$

$y = t^2 \qquad x = t$

$dy = 2t \qquad dx = 1$

$$\int_0^2 t^2 - 2t^2$$

$$\int_0^2 -t^2$$

$$-4$$

29) $\langle 0,0,0 \rangle + t\langle 1,4,4 \rangle$

$r(t) = \langle t, 4t, 4t \rangle$

$r'(t) = \langle 1, 4, 4 \rangle$

$$-3t + 0 \cdot + 16t$$

$$\int_0^1 13t$$

$$\frac{13t^2}{2}$$

$$\frac{13}{2}$$

31) $\int \dfrac{-y\,dx + x\,dy}{x^2+y^2}$ $\quad$ $(\underline{1},0)$ $(\underline{0},1)$

$$P + t(Q-P)$$

$$(1,0)+t(-1,1)$$

$$r(t) = \langle 1-t\,,\, t\rangle$$

$$r'(t) = \langle -1, 1\rangle$$

$$\frac{-t \;+ 1-t}{1+t^2-2t+t^2}$$

$1+2-2$

$$\int_0^1 \frac{1-2t}{1+2t^2-2t}$$

$u = 1+2t^2-2t$

$du = 4t-2$

$= -2(1-2t)$

$$\int_1^1 -\frac{du}{2u}$$

$$= 0$$

The blue path from P to Q

$P \to A$

$(0,0,0) \to (0,0,1)$

$P + t(A - P)$

$t(0,0,1)$ $\qquad r'(t) = \langle 0,0,1 \rangle$

$\langle 0,0,t \rangle$

$f(t) = \langle e^t, e^0, e^0 \rangle$

$= \langle e^t, 1, 1 \rangle$

$f \cdot dr = \langle 0,0,1 \rangle$

$A \to B$

$(0,0,1) + t\langle 0,1,0\rangle$

$\langle 0, t, 1\rangle$ $\qquad r'(t) = \langle 0,1,0\rangle$

$B \to Q$

$(0,1,1) + t(-1,0,0)$

$\langle -t, 1, 1\rangle$ $\qquad r'(t) = \langle -1,0,0\rangle$

$f(t) = \langle e^{-t}, e^{0}, e^{1}\rangle$

$= \langle e^{-t}, 1, e\rangle$

$f(t) \cdot dr = \langle -e^{-t}, 0, 0\rangle$

$$= \int_0^1 1 - e^{-t}$$

$$\left[t - e^{-t}\right]_0^1$$

$$= \cancel{1} - e^{-1} - [\cancel{-1}]$$

$$= -\frac{1}{e}$$

16.3

1) $f = xy\sin(yz)$

$\int \nabla f \cdot dr = f(b) - f(a)$

$= f(1,1,\pi) - f(0,0,0)$

$= \sin\pi - 0 = 0$

19) $f = x^2y - z$

$r_1 = \langle t, t, 0 \rangle$ $\quad$ $r_2 = \langle t, t^2, 0 \rangle$

$0 \le t \le 1$ $\quad$ $0 \le t \le 1$

P $\langle 0,0,0 \rangle$ $\quad$ Q $\langle 1,1,0 \rangle$ $\quad$ P $\langle 0,0,0 \rangle$ $\quad$ Q $\langle 1,1,0 \rangle$

Step 1:- Find $F = \nabla f$

$F = \langle f_x, f_y, f_z \rangle$

$= \langle 2xy, x^2, -1 \rangle$

first curve

$r_1 \langle t, t, 0 \rangle$

$r_1'(t) = \langle 1, 1, 0 \rangle$

$$\int F \cdot \langle dx, dy, dz \rangle$$

$$\int_0^1 \langle 2t^2, t^2, -1 \rangle \cdot \langle 1, 1, 0 \rangle \, dt$$

$$2t^2 + t^2$$

$$\int_0^1 3t^2$$

$$\left[\frac{3t^3}{3} \right]_0^1 = 1$$

second curve

$r_2(t) = \langle t, t^2, 0 \rangle$

$r_2'(t) = \langle 1, 2t, 0 \rangle$

$$\int_0^1 \langle 2t^3, t^4, 0 \rangle \cdot \langle 1, 2t, 0 \rangle \, dt$$

$$\int_0^1 \quad 2t^3 + 2t^5$$

$$\left[2\left(\frac{t^4}{4} + \frac{t^6}{6} \right) \right]_0^1$$

$$\frac{\cancel{2}}{\cancel{4}_2} + \frac{\cancel{2}^1}{\cancel{6}_3} = \frac{5}{6}$$

Curve 1 $\qquad f = x^2 y - z$

$f(b) - f(a)$

$f(1,1,0) - f(0,0,0)$

$1 - 0$

$= 1$

Curve 2

$f(1,1,0) - f(0,0,0)$

$= 1$

3) $F(x,y) = \langle 3, 6y \rangle$

$f(x,y) = 3x + 3y^2$

$r(t) = \langle t, 2t^{-1}, \rangle$

$1 \le t \le 4 \qquad r'(t) = \langle 1, -\frac{4}{t^2} \rangle$

$$\int_1^4 \langle 3, \frac{12}{t} \rangle \langle dx, dy \rangle$$

$$\int_1^4 3 - \frac{12(4)}{t^3}$$

$$\left[3t + \frac{12(2)}{t^2} \right]_1^4$$

$$12 + \frac{\overset{3}{\cancel{12}}(\cancel{2})}{\cancel{4}\times\cancel{4}\,2} - 3 - 12(2)$$

$$12 + \frac{3}{2} - 3 - 24 = -15 + \frac{3}{2}$$
$$= -27/2$$

5) $r(t) = (t^2, t^3, t-1)$

$r'(t) = (2t, 3t^2, 1)$

$F(x,y,z) = ye^z i + xe^z j + xye^z k$

$$= (t^3 e^{t-1}) i + (t^2 e^{t-1}) j + (t^5 e^{t-1}) k$$

$F \cdot r'(t) = 2t^4 \cdot e^{t-1} + 3t^4 e^{t-1} +$
$t^5 e^{t-1}$

$$\int_1^2 e^{t-1}(5t^4 + t^5)$$

$$= 32e - 1$$

17) $r(t) = \left(t^2, \sin\left(\frac{\pi t}{4}\right), e^{t^2-2t}\right)$

$$r'(t) = \left(2t, \frac{\pi}{4}\cos\left(\frac{\pi t}{4}\right), e^{t^2-2t} \cdot 2t-2\right.$$

$$\int_0^2 \left(2t^2 \sin\left(\frac{\pi t}{4}\right) e^{t^2-2t}(2t) + t^4 (e^{t^2-2t}) \frac{\pi}{4}\right.$$

$$\left. \cos\left(\frac{\pi t}{4}\right) + t^4 \sin\left(\frac{\pi t}{4}\right) 4t \right) dt$$