(9) $f(x,y) = \sqrt{1+9xy}$

$\boxed{y = x^3}$ $\boxed{0 \le x \le 1}$

$ds = \sqrt{}$

$x = t$

$y = t^3$

$x't = 1$

$y't = 3t^2$

$\cancel{\int_0^1} \sqrt{1+9x^4}\ \cancel{ds}$

(9) $x = t$ $x' = 1$

$y = t^3$ $y' = 3t^2$

$\cancel{f(x) =}$

$f(x(t), y(t)) = \sqrt{1+9(t^4)}$

$ds = \sqrt{(1)^2 + (3t^2)^2}$

$= \sqrt{1+9t^4}$

$\sqrt{1+9t^3} = \cancel{2} u$

$\boxed{\int_0^1 \sqrt{1+9t^4}\ (\sqrt{1+9t^4})}$

$\int_0^1 (1+9t^4)\ \cancel{(1+9t^3)}$

$\int_0^1 (1+9t^4)\, dt$

$t + \dfrac{9t^5}{5} \Big|_0^1$

$1 + \dfrac{9}{5}$

$= \dfrac{14}{\cancel{5}}\ 2.8$

$= 2.8$

(11) $f(x,y,z) = z^2$

$r(t) = (2t, 3t, 4t)$ for $0 \le t \le 2$

$f(x(t), y(t), z(t)) = 16t^2$

$ds = \sqrt{29}$

$$\int_0^2 16t^2 \sqrt{29}$$

$$\sqrt{29}(16)\left(\frac{t^3}{3}\right)\Big|_0^2$$

$$\boxed{\sqrt{29}(16)\left(\frac{8}{3}\right)}$$

$ds = \cancel{\sqrt{(2t)^2 + (3t)}}$

$= \sqrt{(2)^2 + (3)^2 + (4)^2}$

$= \sqrt{4+9+16}$

$= \sqrt{13+16}$

$= \sqrt{29}$

e

2

Homework:

13 $\boxed{f(x,y,z) = xe^{z^2}}$

I. $(0,0,1)$ to $(0,2,0)$

$\langle P,Q,R\rangle = (0,2,-1)t + (0,0,1)$

$\langle 0, 2t, 1-t\rangle = \langle P,Q,R\rangle$

$x(t) = 0 \qquad dx = 0$

$y(t) = 2t \qquad dy = 2dt$

$z(t) = 1-t \qquad dz = -1\,dt$

$$\int_0^1 P\,dx + Q\,dy + R\,dz$$

$$= \cancel{\int_0^1 0.0 + 2t(2) + (1-t)(-1)}$$

$\int_0^1 0e = \underline{\underline{0}}$

II $(0,2,0)$ to $(1,1,1)$

$\langle 1,-1,1\rangle t + (0,2,0)$

$\langle 1t, 2-t, t\rangle$

$x(t) = t$

$y(t) = 2-t$

$z(t) = t$

$ds = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}$

$= \sqrt{(1)^2 + (-1)^2 + (1)^2}$

$\boxed{= \sqrt{3} \quad dt}$

$\int_0^1 te^{t^2}(\sqrt{3})\,dt$

$t^2 = u$

$2t\,dt = du$

$t\,dt = \frac{1}{2}du$

$\frac{\sqrt{3}}{2}\int e^u\,du$

$\frac{\sqrt{3}}{2} e^u \Big|_0^1$

$\frac{\sqrt{3}}{2}(e^{t^2})\Big|_0^1$

$\frac{\sqrt{3}}{2} e^1 - e^0$

$\boxed{\frac{\sqrt{3}}{2}(e^1 - 1)}$

(17) $\int_C 1\, ds$ $r(t) = (4t, -3t, 12t)$ $\boxed{2 \le t \le 5}$

$x(t) = 4t$ $dx = 4\,dt$

$y(t) = -3t$ $dy = -3\,dt$

$z(t) = 12t$ $dz = 12\,dt$

$ds = \sqrt{(4)^2 + (-3)^2 + (12)^2}$

$= \sqrt{16 + 9 + 144}$

$= \sqrt{169}$

$= \underline{13}$

$13\int_2^5 dt$

$13t \Big|_2^5$

$= 13(3)$

$= \underline{39}$

$\int_C y\,dx - x\,dy$ parabola $y = x^2$ $\boxed{0 \le x \le 2}$

$\boxed{\begin{aligned} x &= t \\ y &= t^2 \end{aligned}}$

$\cancel{t^2 = t}$

$\cancel{t^2 - t = 0}$

$\cancel{t(t-1) = 0}$..

$\cancel{t}$

$\cancel{dx = dt}$

$dy = 2t\,dt$

$\int_0^2 (t^2)\,dt - (t)(2t)\,dt$

$\int_0^2 t^2 - 2t^2\,dt$

$\int_0^2 -t^2\,dt$

$\frac{-t^3}{3}\Big|_0^2$

$= \boxed{\frac{-8}{3}}$

29) $\int (x-y)\,dx + (y-z)\,dy + z\,dz$ $(0,0,0)$ to $(1,4,4)$

$\langle 1,4,4\rangle t + \langle 0,0,0\rangle$

$r(t) = \langle 1t, 4t, 4t\rangle$

$x(t) = t$ $\cancel{x'(t)} =$ $\boxed{dx = 1 \quad dy = 4 \quad dz = 4}$

$y(t) = 4t$

$z(t) = 4t$

$\int (-3t)1 + (0)\,dy + (4t)(4)$

$= \int \cancel{(-3t)}\ 16t - 3t$

$\int_0^1 13t$

$\frac{13t^2}{2}\Big|_0^1$

$\boxed{\frac{13}{2}}$

31) $\int_C \frac{-y\,dx + x\,dy}{x^2+y^2}$ ~~(0,1)~~ $(1,0)$ to $(0,1)$

$\langle -1,1\rangle t + \langle 1,0\rangle$

$r(t) = \langle 1-t, t\rangle$

$x(t) = (1-t)$ $x'(t) = -1\,dt$

$y(t) = t$ $y'(t) = 1\,dt$

$\int_C \frac{-t(-1) + (1-t)}{(1-t)^2 + t^2}$

$\int_C \frac{\cancel{t} + 1 - \cancel{t}}{1^2 + t^2 - 2t + t^2} = \int_C \frac{1}{1+2t^2-2t}$

$(1-\sqrt{2}t)^2$

$\int_C (1+2t^2-2t)^{-1}$

$\langle 0, 1, 0\rangle t + \langle 0, 0, 1\rangle$ Page No. $\langle -1, 0,$

$\langle 0, t, 1\rangle$ Date $+$

$= \langle -t,$

35 $\boxed{P \text{ to } Q}$

$F(x,y,z) = \langle e^z, e^{x-y}, e^y\rangle$

$\boxed{\langle 0, 0, t\rangle}$ $\langle 0, t, 1\rangle$ $\boxed{\langle -t, 1, 1\rangle}$

$\cdot dz$ $\cdot dy$ $\cdot dx$

$x(t) = e^{t} \cdot dz \quad y(t) = e^{0-t} \cdot dy$

$$\int_0^1 P\,dx + Q\,dy + R\,dz$$

$$\int_0^1 (e^1)(-1) + e^{(-t)}(1) + e^0(1)$$

$$\int_0^1 -e^1 + e^{-t} + 1$$

$$= \int_0^1 1 - e + e^{-t}\,dt$$

$$= t - et + \frac{e^{-t}}{-1}\Big|_0^1$$

$$1 - e + \frac{e^{-1}}{-1} - \frac{e^0}{-1}$$

$$= 1 - e - \frac{1}{e} + 1$$

$$\boxed{= 2 - e - \frac{1}{e}}$$

$\cancel{= 2}$

e^{-x}

$-e - \frac{1}{e}$

1, 3, 5, 7, 9, 11, 13, 15, 17, 19

Homework 16.2

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(1) $f(x,y,z) = xy\sin(yz)$ $\boxed{F = \nabla f}$

$$\int_C F \cdot dr = f(b) - f(a)$$

$$= f(1,1,\pi) - f(0,0,0)$$

$$= \sin(\pi) - 0$$

$$= 0 - 0$$

$$= 0$$

$\frac{}{t} \quad -\left(\frac{2}{t^2}\right)$

$=$

(3) $r(t) = \langle t, \frac{2}{t} \rangle$

$x(t) = t \qquad x'(t) = 1dt$

$y(t) = \frac{2}{t} \qquad y'(t) = \frac{-2}{t^2}dt$

$\int \langle 3, 6\left(\frac{2}{t}\right) \rangle \langle 1dt, \frac{-2}{t^2}dt \rangle$

$= \int_1^4 \langle \cancel{4}3 + \frac{12}{t} \cdot \left(-\frac{2}{t^2}\right) \rangle$

$\int_1^4 3 + \left(-\frac{24}{t^3}\right)$

$3\cancel{t} + \frac{\cancel{24}^{12} t^{-2}}{\cancel{+2}}$ $\qquad -3+1$

$3t + \frac{12}{t^2} \Big|_1^4$

$= 3(4) + \left(\frac{12}{16}\right) - (3 + 12)$

$= 12 + \frac{12}{16} - 15$

$= -3 + \frac{\cancel{12}\,\cancel{6}\,3}{\cancel{16}\,\cancel{8}\,4}$

$= -3 + \frac{3}{4}$

$= \frac{-12+3}{4}$

$= \boxed{\frac{-9}{4}}$

(5) $F(x,y,z) = ye^{z}\hat{i} + xe^{z}\hat{j} + xye^{z}\hat{k}$

$f(x,y,z) = xye^{z}$

$r(t) = (t^2, t^3, t-1) \quad 1 \le t \le 2$

$x(t) = t^2, \quad dx = 2t\,dt$

$y(t) = t^3 \quad dy = 3t^2\,dt$

$z(t) = t-1 \quad dz = dt$

$$\cancel{\int_1^2 \langle t^3 . e^{t-1}, t^2 e^{t-1}, t^5 e^{t-1} \rangle \langle 2t, 3t^2, 1 \rangle}$$

$$\cancel{\int_1^2 2t^4 e^{t-1} + 3t^4 e^{t-1} + t^5 e^{t-1}}$$

$$\boxed{\begin{aligned} r(2) &= (4,8,1) \\ r(1) &= (1,1,0) \end{aligned}}$$

$f(4,8,1) - f(1,1,0)$

$f(x,y,z) = xye^{z}$

$= 32e^1 - 1e^0$

$= \boxed{32e - 1}$

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(1) $\boxed{F = y^2\hat{i} + (2xy + e^2)\hat{j} + ye^2\hat{k}}$

$f_x = y^2$

$f = \int y^2$

$f = \frac{y^3}{3} + g(y,z)$ $= \frac{y^3}{3} + xy^2 + ye^2 - \frac{y^3}{3} + h(z)$

ye^x
$= ye^x$

$f_y = 2xy + e^2$

$\frac{1}{3}(3y^2) = 2xy + e^2$

$y^2 + g_-(y,z) = 2xy + e^2$

$g_-(y,z) = \int 2xy + e^2 - y^2 \, dy$

$g(y,z) = \frac{2xy^2}{2} + ye^2 - \frac{y^3}{3} + h(z)$

$f_z = ye^2$

$ye^2 + h'(z) = ye^2$

$h'(z) = 0$

$h(z) = \int 0 \, dz$

$\boxed{h(z) = 0 + C}$

$f(x,y,z) = \boxed{\cancel{\frac{y^3}{3}} + xy^2 + ye^2 \cancel{- \frac{y^3}{3}} + 0 + C}$

4sec² x

(13) $F = \langle z\sec^2 x, z, y + \tan x \rangle$

$f_x = z\sec^2 x$

$f = \int z \sec^2 x \, dx$

$= z\tan x + g(y,z)$

$f_y = z$

$\cancel{\tan x} + g'(y,z) = z$

$g'(y,z) = z \cancel{- \tan x}$

$g(y,z) = \int z \cancel{- \tan x} \, dy$

$g(y,z) = zy \cancel{- y\tan x} + h(z)$

$f = z\tan x + zy \cancel{- y\tan x} + h(z)$

$f_z = y + \tan x$

$\cancel{\tan x} + \cancel{y} + h'(z) = \cancel{y} + \cancel{\tan x}$

$f(x,y,z) = \boxed{z\tan x + zy \cancel{- y\tan z}}$

(15) $F = \langle 2xy + 5, x^2 - 4z, -4y \rangle$

$f_x = 2xy + 5$

$yx^2 + 5x - 4zy$

$f = \int 2xy + 5 \, dx$

$f = 2y\left(\frac{x^2}{2}\right) + 5x + g(y,z)$

$f = yx^2 + 5x + g(y,z)$

$f_y = x^2 + g_y(y,z) = x^2 - 4z$

$\cancel{x^2} + g_{-}(y,z) = \cancel{x^2} - 4z$

$g = g(y,z) = \int -4z\,dy$

$\boxed{g(y,z) = -4zy} + h(z)$

$f_{-z} = -4yz$

$\cancel{-4xy} + h(z) = \cancel{4yz}$

$\boxed{h(z) = 0}$

$\boxed{f(x,y,z) = yx^2 + 5x - 4zy}$

(17) $\int_{C}^{P} 2xyz\,dx + x^2 z\,dy + x^2 y\,dz$

$r(t) = \left(t^2, \sin\left(\frac{\pi t}{4}\right), e^{t^2-2t}\right)$ for $0 \le t \le 2$

~~x(t)~~

$f_{-x} = 2xyz$

$f = \int 2xyz\,dx$

$\boxed{f = yzx^2}$

$f = \cancel{2}yz\left(\frac{x^2}{\cancel{2}}\right) + g(y,z)$

$\boxed{f = yzx^2 + g(y,z)}$

$f_{-y} = zx^2 + g_{-}(y,z) = x^2 z$

$h'(z) = 0$

$g_{-}(y,z) = x^2 z - zx^2$

$\boxed{h(z) = 0 + c}$

$g(y,z) = 0 + h(z)$

$f = yzx^2 \cancel{+ g(y,z)} + h(z)$

~~$f =$~~ $f_{-z} = x^2 y$

$y\cancel{x^2} + h'(z) = x^2\cancel{y}$

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$r(2) = (4, \sin(\frac{\pi}{2}), 1)$

$r(0) = (0, 0, 1)$

$f = yzx^2$

$f(2) - f(0) = 16(1)(\sin\frac{\pi}{2}) - 0$

$= 16 - 0$

$= 16$

$\frac{2t^4}{4\,2}$

$\frac{4t^6}{6\,3}$

(19) $f = x^2y - z$

$r_1 = \langle t, t, 0 \rangle \quad 0 \le t \le 1$

$r_2 = \langle t, t^2, 0 \rangle \quad 0 \le t \le 1$

$F = \nabla f$

$= \langle 2xy, x^2, -1 \rangle$

$\int \langle 2t^2, t^2, -1 \rangle \langle 1, 1, 0 \rangle$

$= \int_0^1 2t^2 + t^2 \, dt$

$= \frac{2t^3}{3} + \frac{t^3}{3} \Big|_0^1$

$= \frac{1}{2} + \frac{1}{4}$

$= \frac{3}{4}$ $\quad \frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$

$t^4 \cdot 2t$

$\int \langle 2t^3, t^4, -0 \rangle \langle 1, 2t, 0 \rangle$

$\int_0^1 2t^3 + 2t^5$

$\frac{2t^4}{4} + \frac{2t^6}{6} \Big|_0^1$

$= \frac{2}{5} + \frac{1}{7}$ $\quad \frac{1}{2} + \frac{1}{3} = \frac{3+}{6}$

$= \frac{3}{5} = \frac{14+5}{35} = \frac{19}{35}$