3. 15.3

$f(x,y,z) = xe^{y-2z} \quad 0 \le x \le 2 \quad 0 \le y \le 1 \quad 0 \le z \le 1$

$\int_0^2\int_0^1\int_0^1 xe^{y-2z}\,dz\,dy\,dx$

$x\int_0^1 \cancel{x} e^{y-2z}\,dz$

$= x\cdot(-\frac{1}{2}e^{y-2z})\Big|_0^1$

$= -\frac{1}{2}xe^{y-2} + \frac{1}{2}xe^{y}$

$\int_0^1 -\frac{1}{2}xe^{y-2} + \frac{1}{2}xe^{y}\,dy$

$= \frac{1}{2}x\int -e^{y-2} + e^{y}\,dy$

$= \frac{1}{2}x\cdot(-e^{y-2}) + e^{y})\Big|_0^1$

$= \frac{1}{2}x\cdot(-e^{-1}+e^{1}) - \frac{1}{2}x(-e^{-2}+1)$

$= \frac{1}{2}x(-e^{-1}+e^{1}+e^{-2}-1)$

$\int_0^2 \frac{1}{2}x(-e^{-1}+e^{1}+e^{-2}-1)\,dx$

$= \frac{1}{2}(-e^{-1}+e+e^{-2}-1)\int_0^2 x\,dx$

$= \frac{1}{2}(-e^{-1}+e+e^{-2}-1)\cdot\frac{x^2}{2}\Big|_0^2$

$= \frac{1}{2}(-e^{-1}+e+e^{-2}-1)\cdot 2$

$= -e^{-1}+e+e^{-2}-1$

5. $f(x,y,z) = (x-y)(y-z) \quad [0,1]\times[0,3]\times[0,3]$

$\int_0^3 (x-y)(y-z)\,dz$

$= (x-y)(yz - \frac{z^2}{2})\Big|_0^3$

$= (x-y)(3y - \frac{9}{2})$

$\int_0^3 (x-y)(3y-\frac{9}{2})\,dy$

$= 3x\frac{y^2}{2} - \frac{9}{2}xy - \frac{3y^3}{3} + \frac{9}{2}\frac{y^2}{2}\Big|_0^3$

$= -\frac{27}{4}$

$\int_0^1 -\frac{27}{4}\,dx$

$= -\frac{27}{4}x = -\frac{27}{4}\times 1 = -\frac{27}{4}$

7. $f(x,y,z) = (x+z)^3 \quad [0,a]\times[0,b]\times[0,c]$

$\int_0^c (x+z)^3\,dz$

$= \frac{(x+z)^4}{4}\Big|_0^c = \frac{(x+c)^4}{4} - \frac{x^4}{4}$

$\int_0^b \frac{(x+c)^4}{4} - \frac{x^4}{4}\,dy = \frac{y(x+c)^4}{4} - \frac{yx^4}{4}\Big|_0^b$

$= \frac{b(x+c)^4}{4} - \frac{bx^4}{4}$

$\int_0^a \frac{b(x+c)^4}{4}\,dx - \int_0^a \frac{bx^4}{4}\,dx.$

$= \frac{b}{4}\cdot\frac{(x+c)^5}{5} - \frac{b}{4}\cdot\frac{x^5}{5}\Big|_0^a$

$= \frac{b(a+c)^5}{20} - \frac{ba^5}{20} - \frac{bc^5}{20}$

$= \frac{b}{20}(a+c)^5 - a^5 - c^5$

9. $f(x,y,z)=x+y$. $y\le z\le x$ $0\le y\le x$ $0\le x\le 1$

$\int_y^x x+y\,dz$

$=(x+y)z\Big|_y^x$

$=(x+y)x-(x+y)y$

$=(x+y)(x-y)$

$\int_0^x x^2-y^2\,dy$.

$=x^2y-\frac{y^3}{3}\Big|_0^x$

$=x^3-\frac{x^3}{3}$

$\int_0^1 x^3-\frac{x^3}{3}\,dx$

$=\frac{x^4}{4}-\frac{x^4}{12}\Big|_0^1$

$=\frac{1}{4}-\frac{1}{12}=\frac{1}{6}$

11. $f(x,y,z)=xyz$ $0\le z\le 1$ $0\le y\le\sqrt{1-x^2}$

$0\le x\le 1$

$\int_0^1 xyz\,dz$

$=xy\int_0^1 z\,dz=xy\cdot\frac{z^2}{2}\Big|_0^1$

$=\frac{1}{2}xy$

$\int_0^{\sqrt{1-x^2}}\frac{1}{2}xy\,dy=\frac{1}{2}x\int_0^{\sqrt{1-x^2}}y\,dy$

$=\frac{1}{2}x\,\frac{y^2}{2}\Big|_0^{\sqrt{1-x^2}}$

$=\frac{1}{2}x\cdot\frac{1-x^2}{2}$

$\int_0^1\frac{x-x^3}{4}\,dx$

$=\frac{1}{16}$

13. $f(x,y,z)=e^z$ $x+y+z\le 1$ $x\ge 0$ $y\ge 0$ $z\ge 0$

~~$z=1$ $0\le z\le 1$~~ $z=1-x-y$ $0\le z\le 1-x-y$.

$z=0$ $x+y=1$ $y=1-x$ $0\le y\le 1-x$

$0\le x\le 1$

$\int_0^{1-x-y}e^z\,dz$

$=e^z\Big|_0^{1-x-y}$

$=e^{1-x-y}-1$

$\int_0^{1-x}e^{1-x-y}-1\,dy$

$=-e^{1-x-y}-y\Big|_0^{1-x}$

$=-e^{1-x-1+x}-1+x+e^{1-x}$

$=-e^0-1+x+e^{1-x}$

$=-2+x+e^{1-x}$

$\int_0^1 -2+x+e^{1-x}\,dx$

$-2x+\frac{x^2}{2}-e^{1-x}\Big|_0^1$

$=-2+\frac{1}{2}-e^0+e$

$=-2+\frac{1}{2}-1+e$

$=e-\frac{5}{2}$

15. $z=\sqrt{9-x^2-y^2}$

$0\le z\le\sqrt{9-x^2-y^2}$

$0\le y\le x$

$0\le x\le 1$

$\int_0^{\sqrt{9-x^2-y^2}}z\,dz$

$=\frac{z^2}{2}\Big|_0^{\sqrt{9-x^2-y^2}}=\frac{9-x^2-y^2}{2}$

$\int_0^x\frac{9-x^2-y^2}{2}\,dy$

$=\frac{9}{2}y-\frac{x^2y}{2}-\frac{y^3}{6}\Big|_0^x$

$=\frac{9}{2}x-\frac{x^3}{2}-\frac{x^3}{6}$

$\int_0^1\frac{9x}{2}-\frac{x^3}{2}-\frac{x^3}{6}\Big|_0^1$

$=\frac{9}{2}\cdot\frac{x^2}{2}-\frac{1}{2}\cdot\frac{x^4}{4}-\frac{x^4}{24}\Big|_0^1$

$=\frac{9x^2}{4}-\frac{x^4}{8}-\frac{x^4}{24}\Big|_0^1$

$=\frac{9}{4}-\frac{1}{8}-\frac{1}{24}=\frac{25}{12}$

17. I can't find the region of x.

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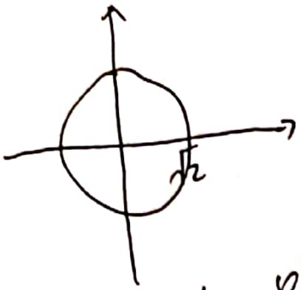
15.4

1. $f(x,y)=\sqrt{x^2+y^2}$, $x^2+y^2\le 2$

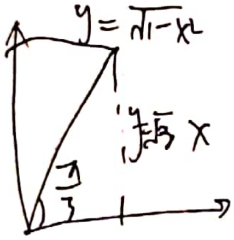


$\iint_D \sqrt{x^2+y^2}\,d\sigma=\frac{4\sqrt{2}\pi}{3}$

$0\le\theta\le2\pi$ $0\le r\le\sqrt{2}$

$\iint_D r\cdot r\,dr\,d\theta$

4. $\int_0^{\frac{1}{2}}\int_{\sqrt{3}x}^{\sqrt{1-x^2}} x\,dy\,dx$



$\int_0^{\frac{1}{2}}\int_{\sqrt{3}x}^{\sqrt{1-x^2}} x\,dy\,dx=0.045$

27. $f(x,y,z)=x^2+y^2$ $x^2+y^2\le 9$ $0\le z\le 5$

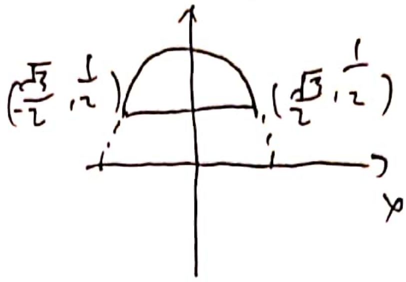
$\int_0^{2\pi}\int_0^{3}\int_0^{5} r^3\,dz\,dr\,d\theta$

$\int_0^5 r^3dz=r^3z\Big|_0^5=5r^3$

$\int_0^3 5r^3dr=\frac{5r^4}{4}\Big|_0^3=\frac{405}{4}$

$\int_0^{2\pi}\frac{405}{4}d\theta=\frac{405}{4}\cdot2\pi=\frac{405\pi}{2}$

5. $f(x,y)=y(x^2+y^2)^{-1}$ $y\ge\frac{1}{2}$ $x^2+y^2\le1$



$\iint_D y(x^2+y^2)^{-1}\,d\sigma=\sqrt{3}-\frac{\pi}{3}.$

19. $f(x,y)=x-y$; $x^2+y^2\le1$ $x+y\ge1$

$r=0..1$ $0\le\theta\le\frac{\pi}{2}$

31. $f(x,y,z)=z$ $x^2+y^2\le z\le 9$.

$0\le r\le3$ $0\le\theta\le2\pi$ $0\le z\le2\pi$

$\int_0^{2\pi} z\,r\,dz=\frac{z^2}{2}r\Big|_0^{2\pi}$

$=\frac{4\pi^2}{2}r=2\pi^2r$

$\int_0^3 2\pi^2 r\,dr=2\pi^2\cdot\frac{r^2}{2}\Big|_0^3$

$=2\pi^2\frac{9}{2}=9\pi^2$

$\int_0^{2\pi}9\pi^2\,d\theta$

$=9\pi^2\cdot2\pi=18\pi^3$.

47.

$f(x,y,z)= x^2+y^2 \quad \rho\le 1$

$0\le\theta\le 2\pi \quad 0\le\rho\le 1 \quad 0\le\phi\le\pi$

$$\int_0^{\pi}\int_0^{2\pi}\int_0^1 (\rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta)\,\rho^2\sin\phi\, d\rho\, d\theta\, d\phi.$$

$$=\int_0^{\pi}\int_0^{2\pi}\int_0^1 \rho^4\sin^3\phi\cos^2\theta+\rho^4\sin^3\phi\sin^2\theta\, d\rho\, d\theta\, d\phi.$$

$$=\frac{8}{15}\pi$$

57. $f(x,y,z)=z$

$z=\rho\cos\phi$

$$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{3}}\int_1^2 \rho\cos\phi\,\rho^2\sin\phi\, d\rho\, d\theta\, d\phi$$

$$=\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{3}}\int_1^2 \rho^3\cos\phi\sin\phi\, d\rho\, d\theta\, d\phi$$

$$\int_1^2 \rho^3\, d\rho=\frac{\rho^4}{4}\Big|_1^2=4-\frac{1}{4}=\frac{15}{4}$$

$$\int_0^{\frac{\pi}{3}} d\theta=\frac{\pi}{3}$$

$$\int_0^{\frac{\pi}{2}}\cos\phi\sin\phi\, d\phi=\frac{1}{2}$$

$$\frac{\cancel{15}\,5}{4}\times\frac{\pi}{\cancel{3}}\times\frac{1}{2}=\frac{5\pi}{8}$$