Math 251 Shaun Goda Section 23 HW#8

January February March April May June July August September October November December

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

15.3:

3) $\int_0^2\int_0^1\int_0^1 x e^{y-2z}\,dz\,dy\,dx \Rightarrow \int_0^1 x e^{y-2z}\,dz = \left| \frac{-x e^{y-2z}}{2} \right|_0^1 = \left(\frac{1-e^{-2}}{2}\right) x e^{y}$

$$\int_0^1 \left(\frac{1-e^{-2}}{2} x e^{y}\right) dy = \frac{(e^2-1)(e-1)e^{-2}x}{2}$$

$$\int_0^2 \left(\frac{(e^2-1)(e-1)e^{-2}\cdot x}{2}\right) dx = \boxed{(e^2-1)(e-1)e^{-2}}$$

5) $\int_0^1\int_0^3\int_0^3 (x-y)(y-z)\,dz\,dy\,dx \Rightarrow \int_0^3 (x-y)(y-z)\,dz = \left| xyz - \frac{xz^2}{2} - y^2 z + \frac{yz^2}{2} \right|_0^3$

$$= -3\left(y^2 - xy + x\frac{9}{2} - y\frac{9}{2}\right)$$

$$-3\int_0^3 \left(y^2 - xy + x\frac{9}{2} - y\frac{9}{2}\right) dy = -3\left| \frac{y^3}{3} - \frac{xy^2}{2} + x y\frac{3}{2} - \frac{y^2}{2} \right|_0^3$$

$$= -\frac{27}{4}$$

$$\int_0^1 -\frac{27}{4}\,dx = \boxed{-\frac{27}{4}}$$

7) $\int_0^a\int_0^b\int_0^c (x+z)^3\,dz\,dy\,dx \Rightarrow \frac{(x+c)^4}{4} - \frac{x^4}{4} = \int_0^c (x+z)^3\,dz$

$$\int_0^b \left(\frac{(x+c)^4}{4} - \frac{x^4}{4}\right) dy = b\left(\frac{(c+x)^4 - x^4}{4}\right)$$

$$\int_0^a b\left(\frac{(c+x)^4 - x^4}{4}\right) dx = \boxed{\frac{b(c^4 a + 2c^3a^2 + 2c^2a^3 + ca^4)}{4}}$$

9) $\int_0^1\int_0^x\int_y^x (x+y)\,dz\,dy\,dx \Rightarrow \int_y^x (x+y)\,dz = \left| xz + yz \right|_y^x = x^2 - y^2$

$$\int_0^x (x^2 - y^2)\,dy = \left| x^2 y - \frac{y^3}{3} \right|_0^x = x^3 - \frac{x^3}{3}$$

$$\int_0^1 \left(x^3 - \frac{x^3}{3}\right) dx = \left| \frac{x^4}{4} - \frac{x^4}{12} \right|_0^1 = \frac{1}{4} - \frac{1}{12} = \boxed{\frac{1}{6}}$$

11) $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^1 xyz\,dz\,dy\,dx \Rightarrow \int_0^1 xyz\,dz = \left|\frac{xyz^2}{2}\right|_0^1 = \frac{xy}{2}$

$\int_0^{\sqrt{1-x^2}} \frac{xy}{2}\,dy = \left|\frac{xy^2}{4}\right|_0^{\sqrt{1-x^2}} = \frac{x\sqrt{1-x^2}}{4}$

$\int_0^1 \frac{x\sqrt{1-x^2}}{4}\,dx = \left|-\frac{1}{12}(1-x^2)^{\frac{3}{2}}\right|_0^1 = \boxed{\frac{1}{12}}$

13) $\int_0^{1-y-z}\int_0^{1-x-z}\int_0^{1-x-y} e^z\,dz\,dy\,dx \Rightarrow \int_0^{1-x-y} e^z\,dz = e^{1-x-y}$

$\int_0^{1-x-z} e^{1-x-y}\,dy = -e^{1-x}(e^{-1+x+z}-1)$

$\int_0^{1-x-y} -e^{1-x}(e^{-1+x+z}-1) = \boxed{-e^{x+y}-e+e^z x-e^z+ye^z}$

Not so sure...

15) $\int_0^1\int_0^x\int_0^{\sqrt{9-x^2-y^2}} z\,dz\,dy\,dx \Rightarrow \int_0^{\sqrt{9-x^2-y^2}} z\,dz = \frac{9-x^2-y^2}{2}$

$\frac{1}{2}\int_0^x (9-x^2-y^2)\,dy = \frac{1}{2}\left|9y-x^2y-\frac{y^3}{3}\right|_0^x = \left(9x-x^3-\frac{x^3}{3}\right)\frac{1}{2}$

$\frac{1}{2}\int_0^1\left(9x-x^3-\frac{x^3}{3}\right)dx = \frac{1}{2}\left|\frac{9x^2}{2}-\frac{x^4}{4}-\frac{x^4}{12}\right|_0^1 = \frac{1}{2}\left(\frac{9}{2}-\frac{1}{4}-\frac{1}{12}\right)$

$= \frac{1}{2}\cdot\frac{25}{6} = \boxed{\frac{25}{12}}$

17) $y^2 = 8-2x^2-y^2 \Rightarrow 2y^2 = 8-2x^2$ $y = \sqrt{4-x^2}$ when $z=0$

$0 = \sqrt{4-x^2}$ $x^2 = 4$ $x = 2$ thus...

$0 \le x \le 2,\ 0 \le y \le \sqrt{4-x^2},\ 0 \le z \le 8-2x^2-y^2$

find $\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{8-2x^2-y^2} x\,dz\,dy\,dx \Rightarrow \int_0^{8-2x^2-y^2} x\,dz =$

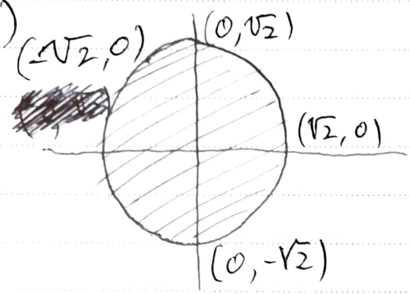
$x(8-2x^2-y^2)$
$= 8x-2x^3-y^2x$

$\int_0^{\sqrt{4-x^2}}(8x-2x^3-y^2x)\,dy = -2x^3\sqrt{-x^2+4}+8x\sqrt{-x^2+4} - x\frac{(-x^2+4)^{\frac{3}{2}}}{3}$

$\int_0^2\left(-2x^3\sqrt{-x^2+4}+8x\sqrt{-x^2+4} - x\frac{\sqrt{(-x^2+4)^3}}{3}\right)dx = \boxed{\frac{32}{3}}$

15.4:

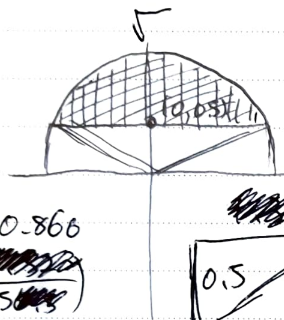
1)



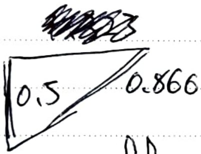
$$\iint_D \sqrt{x^2+y^2}\,dA = \int_0^{2\pi}\int_0^{\sqrt{2}} \sqrt{r^2}\; r\,dr\,d\theta$$

$$= \left|\frac{r^3}{3}\right|_0^{\sqrt{2}} = \frac{2\sqrt{2}}{3},\quad \int_0^{2\pi}\frac{2\sqrt{2}}{3}\,d\theta = \left|\frac{2\sqrt{2}}{3}\theta\right|^{2\pi} = \boxed{\frac{4\pi\sqrt{2}}{3}}$$

5)



$\tan^{-1}\left(\frac{0.866}{0.5}\right) = \frac{\pi}{6}$



$$\iint_D y(x^2+y^2)^{-1}\,dA = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\int_0^1 \sin\theta\,dr\,d\theta \text{ − Area of triangle}$$

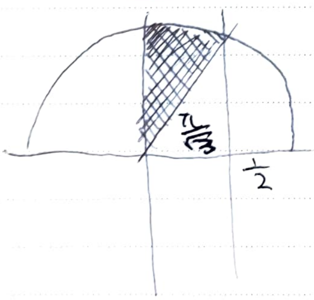
$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\int_0^1 \sin\theta\,dr\,d\theta = \sqrt{3}$$

$$\iint_D y(x+y^2)^{-1}\,dA = \sqrt{3} - 2\int_0^1\int_0^{\sqrt{3}x} y(x^2+y^2)^{-1}\,dy\,dx$$

$$2\int_0^1\int_0^{\sqrt{3}x} y(x^2+y^2)^{-1}\,dy\,dx = \frac{\pi}{3}$$

$$\boxed{\text{Volume} = \sqrt{3} - \frac{\pi}{3} \approx 0.6849}$$

9)



$$\iint_D x\,dA = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\int_0^1 r^2\cos\theta\,dr\,d\theta$$

$$\int_0^1 r^2\cos\theta\,dr = \left|\frac{r^3\cos\theta}{3}\right|_0^1 = \frac{\cos\theta}{3}$$

$$\frac{1}{3}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\cos\theta\,d\theta = \frac{1}{3}\left|\sin\theta\right|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \boxed{\frac{-(\sqrt{3}-2)}{6} \approx 0.04466}$$

19) $\iint_D x - y \, dA = \int_0^{\frac{\pi}{2}}\int_0^1 (r^2\cos\theta - r^2\sin\theta)\, dr\, d\theta - [\text{area below } y = -x + 1]$

$$\int_0^1 (r^2\cos\theta - r^2\sin\theta)dr = \left| \frac{r^3\cos\theta}{3} - \frac{r^3\sin\theta}{3} \right|_0^1 = \frac{\cos\theta}{3} - \frac{\sin\theta}{3}$$

$$\frac{1}{3}\int_0^{\frac{\pi}{2}} (\cos\theta - \sin\theta) d\theta = \frac{1}{3}\left| \sin\theta + \cos\theta \right|_0^{\frac{\pi}{2}} = \frac{1}{3}(1 + 0) = (0)\frac{1}{3} = 0$$

area below $y = -x + 1 \Rightarrow \int_0^1\int_0^{1-x} (x - y)\, dy\, dx$

$$\int_0^{1-x} (x - y) dy = \left| xy - \frac{y^2}{2} \right|_0^{1-x} = x(1-x) - \frac{(1-x)^2}{2}$$

$$\int_0^1 \left(x(1-x) - \frac{(1-x)^2}{2} \right) dx = 0$$

$$\iint_D (x - y) dA = 0 - 0 = \boxed{0}$$

27) $\int_0^{2\pi}\int_0^3\int_0^5 r^3 \, dz\, dr\, d\theta \Rightarrow \int_0^5 r^3 dz = \left| r^3 z \right|_0^5 = 5r^3$

$$\int_0^3 5r^3 dr = \left| \frac{5r^4}{4} \right|_0^3 = \frac{405}{4}$$

$$\int_0^{2\pi} \frac{405}{4} d\theta = \left| \frac{405}{4}\theta \right|_0^{2\pi} = \boxed{\frac{405}{2}\pi = 318.086}$$

31) $\int_0^{2\pi}\int_0^3\int_0^9 z \, dz\, dr\, d\theta \Rightarrow \int_0^9 z\, dz = \left| \frac{z^2}{2} \right|_0^9 = \frac{81}{2}$

$$\int_0^3 \frac{81}{2} dr = \left| \frac{81}{2} r \right|_0^3 = \frac{243}{2}$$

$$\int_0^{2\pi} \frac{243}{2} d\theta = \left| \frac{243}{2}\theta \right|_0^{2\pi} = \boxed{243\pi}$$

47) $\iiint_E (x^2+y^2)\,dV = \int_0^{\pi}\int_0^{2\pi}\int_0^{1} \left((\rho\sin\varphi\cos\theta)^2+(\rho\sin\varphi\sin\theta)^2\right)\rho^2\sin\varphi\,d\rho\,d\theta\,d\varphi$

use calculator... $= \boxed{\frac{8\pi}{15}}$

51) $\iiint_E z\,dV = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{3}}\int_1^{2} \rho^3\sin\phi\cos\phi\,d\rho\,d\theta\,d\phi$

~~use calculator~~ $\Rightarrow \int_1^2 \rho^3\sin\phi\cos\phi\,d\rho = \left|\frac{\rho^4\sin\phi\cos\phi}{4}\right|_1^2 = \frac{15\sin\phi\cos\phi}{4}$

$\frac{15}{4}\int_0^{\frac{\pi}{3}}\sin\phi\cos\phi\,d\theta = \frac{15}{4}\left|\sin\phi\cos\phi\,\theta\right|_0^{\frac{\pi}{3}} = \frac{15}{4}\left(\frac{\pi}{3}\sin\phi\cos\phi\right)$

$\frac{15\pi}{12}\int_0^{\frac{\pi}{2}}\sin\phi\cos\phi\,d\phi = \frac{15\pi}{12}\left|\frac{-\cos^2\phi}{2}\right|_0^{\frac{\pi}{2}} = \boxed{\frac{5\pi}{8}}$