HW due 11/01

15.3: 3, 5, 7, 9, 11, 13, 15, 17

15.4: 1, 5, 9, 19, 27, 31, 47, 51

15.3

3. $\int_0^2\int_0^1\int_0^1 xe^{y-2z}\,dz\,dy\,dx$

$= \left(\int_0^2 x\,dx\right)\left(\int_0^1 e^y\,dy\right)\left(\int_0^1 e^{-2z}\,dz\right)$

$= \left(\frac{x^2}{2}\Big|_0^2\right)\left(e^y\Big|_0^1\right)\left(-\frac{e^{-2z}}{2}\Big|_0^1\right)$

$= (2)(e-1)\left(\frac{e^{-2}+1}{2}\right) = \boxed{(e-1)(e^{-2}+1)}$

5. $\int_0^1\int_0^3\int_0^3 xy - xz - y^2 + yz\,dz\,dy\,dx$

$= xyz - \frac{xz^2}{2} - zy^2 + \frac{yz^2}{2}\Big|_0^3$

$= 3xy - \frac{9x}{2} - 3y^2 + \frac{9y}{2}$

$\int_0^3 3xy - \frac{9x}{2} - 3y^2 + \frac{9y}{2}\,dy = \frac{3xy^2}{2} - \frac{9xy}{2} - y^3 + \frac{9y^2}{4}\Big|_0^3$

$= \frac{27x}{2} - \frac{27x}{2} - 27 + \frac{81}{4} = 6.75$

$\int_0^1 6.75\,dx = 6.75x\Big|_0^1 = \boxed{6.75}$

7. $\int_0^a\int_0^b\int_0^c (x+z)^3\,dz\,dy\,dx$

$= \frac{(x+z)^4}{4}\Big|_0^c = \frac{(x+c)^4}{4} - \frac{x^4}{4}$

$\int_0^b \frac{(x+c)^4}{4} - \frac{x^4}{4}\,dy = \frac{y(x+c)^4}{4} - \frac{yx^4}{4}\Big|_0^b$

$\int_0^a \frac{b(x+c)^4}{4} - \frac{bx^4}{4}\,dx = \frac{(x+c)^5 b}{20} - \frac{bx^5}{20}\Big|_0^a$

$= \frac{(a+c)^5 b}{20} - \frac{ba^5}{20} - \frac{c^5 b}{20} = \boxed{\frac{b}{20}\left((a+c)^5 - a^5 - c^5\right)}$

9. $\int_0^1\int_0^x\int_y^x (x+y)\,dz\,dy\,dx$

$= (x+y)z\Big|_y^x = x(x+y) - y(x+y) = x^2 + xy - xy - y^2$

$\int_0^x x^2 - y^2\,dy = x^2y - \frac{y^3}{3}\Big|_0^x = x^3 - \frac{x^3}{3}$

$\int_0^1 x^3 - \frac{x^3}{3}\,dx = \frac{x^4}{4} - \frac{x^4}{12}\Big|_0^1 = \frac{1}{4} - \frac{1}{12} = \boxed{\frac{1}{6}}$

11. $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^1 xyz\,dz\,dy\,dx$

$= \frac{xyz^2}{2}\Big|_0^1 = \frac{xy}{2}$

$\int_0^{\sqrt{1-x^2}} \frac{xy}{2}\,dy = \frac{xy^2}{4}\Big|_0^{\sqrt{1-x^2}} = \frac{x(1-x^2)}{4} = \frac{x}{4} - \frac{x^3}{4}$

$\int_0^1 \frac{x}{4} - \frac{x^3}{4}\,dx = \frac{x^2}{8} - \frac{x^4}{16}\Big|_0^1 = \frac{1}{8} - \frac{1}{16} = \boxed{\frac{1}{16}}$

13. $\int_0^1\int_0^{1-x}\int_0^{1-x-y} e^z\,dz\,dy\,dx$

$= e^z\Big|_0^{1-x-y} = e^{1-x-y} - 1$

$\int_0^{1-x} e^{1-x-y} - 1\,dy = -e^{1-x-y} - y\Big|_0^{1-x}$

$\int_0^1 x - 2 + e^{1-x}\,dx = \frac{x^2}{2} - 2x - e^{1-x}\Big|_0^1$

$= \frac{1}{2} - 2 - 1 + e = \boxed{-\frac{5}{2} + e}$

15. $\int_0^1\int_0^x\int_0^{\sqrt{9-x^2-y^2}} z\,dz\,dy\,dx$

$= \frac{z^2}{2}\Big|_0^{\sqrt{9-x^2-y^2}} = \frac{9-x^2-y^2}{2}$

$\int_0^x \frac{9}{2} - \frac{x^2}{2} - \frac{y^2}{2}\,dy = \frac{9y}{2} - \frac{x^2y}{2} - \frac{y^3}{6}\Big|_0^x$

$\int_0^1 \frac{9x}{2} - \frac{x^3}{2} - \frac{x^3}{6}\,dx = \frac{9x^2}{4} - \frac{x^4}{8} - \frac{x^4}{24}\Big|_0^1$

$= \frac{9}{4} - \frac{1}{8} - \frac{1}{12} = \frac{50}{24} = \boxed{\frac{25}{12}}$

17. $\int_0^2\int_0^{\sqrt{4-y^2}}\int_{y^2}^{8-2x^2-y^2} x\,dz\,dx\,dy$

$= xz\Big|_{y^2}^{8-2x^2-y^2} = 8x - 2x^3 - y^2x - y^2x$

$\int_0^{\sqrt{4-y^2}} 8x - 2x^3 - 2y^2x\,dx = 4x^2 - \frac{x^4}{2} - x^2y^2\Big|_0^{\sqrt{4-y^2}}$

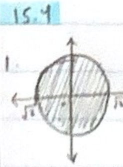
$= 4(4-y^2) - \frac{(4-y^2)^2}{2} - (4-y^2)y^2 = 16 - 4y^2$

$\int_0^2 8 - 4y^2 + \frac{y^4}{2}\,dy = 8y - \frac{4y^3}{3} + \frac{y^5}{10}\Big|_0^2$

$= 16 - \frac{32}{3} + \frac{32}{10} = \boxed{8.53}$

15.4

1.



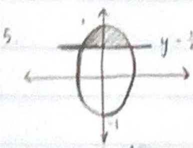
$\int_0^{2\pi}\int_0^{\sqrt{2}} r^2\,dr\,d\theta$

$\frac{r^3}{3}\Big|_0^{\sqrt{2}} = \frac{2\sqrt{2}}{3}$

$\int_0^{2\pi} \frac{2\sqrt{2}}{3}\,d\theta = \frac{2\sqrt{2}}{3}\theta\Big|_0^{2\pi}$

$= \boxed{\frac{4\sqrt{2}\pi}{3}}$

5.



$\int_{\pi/6}^{5\pi/6}\int_{\frac{1}{2\sin\theta}}^{1} \sin\theta\,dr\,d\theta$

$= r\sin\theta\Big|_{\frac{1}{2\sin\theta}}^{1} = \sin\theta - \frac{1}{2}$

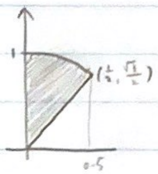
$\int_{\pi/6}^{5\pi/6} \sin\theta - \frac{1}{2}\,d\theta = -\cos\theta - \frac{\theta}{2}\Big|_{\pi/6}^{5\pi/6}$

$= -\cos\frac{5\pi}{6} - \frac{5\pi}{12} + \cos\frac{\pi}{6} + \frac{\pi}{12} = \sqrt{3} - \frac{\pi}{3}$

$\approx \boxed{.685}$

15.4: 9, 19, 27, 31, 47, 51

9.



$$\int_{\pi/3}^{\pi/2}\int_0^1 r^2 \cos\theta \, dr \, d\theta$$

$$= \frac{r^3}{3}\cos\theta \Big|_0^1 = \frac{\cos\theta}{3}$$

$$\int_{\pi/3}^{\pi/2} \frac{\cos\theta}{3} \, d\theta = \frac{\sin\theta}{3}\Big|_{\pi/3}^{\pi/2}$$

$$= \frac{1}{3} - \frac{\sqrt{3}}{6} = \frac{2-\sqrt{3}}{6} = \boxed{.045}$$

19. $$\int_0^{\pi/2}\int_{\frac{1}{\cos\theta+\sin\theta}}^{1} r^2(\cos\theta - \sin\theta) \, dr \, d\theta$$

$$= \frac{r^3}{3}(\cos\theta - \sin\theta)\Big|_{\frac{1}{\cos\theta+\sin\theta}}^{1}$$

$$= \frac{\cos\theta - \sin\theta}{3} - \frac{(\cos\theta - \sin\theta)}{3(\cos\theta+\sin\theta)^3}$$

$$\int_0^{\pi/2} \frac{\cos\theta - \sin\theta}{3} - \frac{\cos\theta - \sin\theta}{3(\cos\theta+\sin\theta)^3} \, d\theta$$

$u = \cos\theta + \sin\theta$

$du = \cos\theta - \sin\theta \, d\theta$

$$= \frac{1}{3}\int_1^1 1 - \frac{1}{u^3} \, du = \boxed{0}$$

27. $$\int_0^{2\pi}\int_0^3\int_0^5 r^3 \, dz \, dr \, d\theta$$

$$= zr^3\Big|_0^5 = 5r^3$$

$$\int_0^3 5r^3 \, dr = \frac{5r^4}{4}\Big|_0^3 = \frac{405}{4}$$

$$\int_0^{2\pi} \frac{405}{4} \, d\theta = \frac{405\theta}{4}\Big|_0^{2\pi} = \frac{405\pi}{2} \approx \boxed{636.173}$$

31. $$\int_0^{2\pi}\int_0^3\int_{r^2}^9 zr \, dz \, dr \, d\theta$$

$$= \frac{z^2 r}{2}\Big|_{r^2}^9 = \frac{81r}{2} - \frac{r^5}{2}$$

$$\int_0^3 \frac{81r}{2} - \frac{r^5}{2} \, dr = \frac{81r^2}{4} - \frac{r^6}{12}\Big|_0^3 = \frac{729}{4} - \frac{729}{12}$$

$$\int_0^{2\pi} \frac{1458}{12} \, d\theta = \frac{1458}{12}\theta\Big|_0^{2\pi} = \boxed{243\pi}$$

47. $$\int_0^{2\pi}\int_0^{\pi}\int_0^1 \rho^4 \sin^3\phi \, d\rho \, d\phi \, d\theta$$

$$= \left(\frac{1}{5}\rho^5\right)_0^1 \left(\frac{\cos^3\phi}{3} - \cos\phi\right)_0^{\pi} (\theta)_0^{2\pi}$$

$$= \left(\frac{1}{5}\right)\left(\frac{4}{3}\right)(2\pi) = \boxed{\frac{8}{15}\pi}$$

51. $$\int_0^{\pi/3}\int_0^{\pi/2}\int_1^2 \rho^3 \cos\phi \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \left(\frac{\rho^4}{4}\right)_1^2 \left(\frac{\sin^2\phi}{2}\right)_0^{\pi/2} (\theta)_0^{\pi/3}$$

$$= \left(4 - \frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{3}\right) = \left(\frac{15}{4}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{3}\right) = \frac{15\pi}{24} = \boxed{\frac{5\pi}{8}}$$