Calc HW Due
11/1

Rahul Paleja

Section 15.3: # 3, 5, 7, 9, 11, 13, 15, 17:

(3)
$$\int_0^1\int_0^1\int_0^2 x e^{y-2z}\,dx\,dy\,dz$$

Inner:
$$e^{y-2z}\int_0^2 x\,dx = \frac{x^2}{2}\Big]_0^2 = 2$$

Middle:
$$2\int_0^1 e^{y-2z}\,dy \qquad u = y-2z \quad du = dy$$
$$= \int_0^1 e^u\,du = e^{y-2z}\Big]_0^1 = 2(e^{1-2z} - e^{-2z})$$

Outer:
$$\int_0^1 2e^{1-2z}\,dz - \int_0^1 2e^{-2z}\,dz$$

$u = 1-2z$
$$\frac{du}{-2} = \frac{-2dz}{-2}$$

$$-\frac{2}{-2}\int_0^1 e^{-2z}\,dz$$

$$-\int_0^1 e^u\,du$$

$u = -2z$
$$\frac{du}{-2} = \frac{-2\,du}{-2}$$

$$= -(e^{1-2z}\Big]_0^1$$

$$\int_0^1 e^u\,du$$

$$-(e^{-1} - e^0)$$

$$= -\frac{1}{e} + 1$$

$$= e^{-2z}\Big)_0^1 = e^{-2} - e^0$$
$$= \frac{1}{e^2} - 1$$

$$\left(\frac{e}{e}\right) - \frac{1}{e} + \cancel{1} + \frac{1}{e^2} \cancel{-1} = \boxed{\frac{1-e}{e^2}}$$

⑤ $f(x,y,z) = (x-y)(y-z)$ $[0,1] \times [0,3] \times [0,3]$

$= xy - y^2 - xz + yz$

$$\int_0^3\int_0^3\int_0^1 xy - y^2 - xz + yz \, dx\, dy\, dz$$

Inner:

$$\int_0^1 xy - y^2 - xz + yz \, dx$$

$$\frac{x^2}{2}y - xy^2 - \frac{x^2}{2}z + xyz \Big]_0^1 = \frac{y}{2} - y^2 - \frac{z}{2} + yz$$

Middle:

$$\int_0^3 \frac{y}{2} - y^2 - \frac{z}{2} + yz \, dy = \frac{y^2}{4} - \frac{y^3}{3} - \frac{yz}{2} + \frac{y^2}{2}z \Big]_0^3$$

$$= \left(\frac{3}{3}\right)\frac{9}{4} - \frac{27}{3}\left(\frac{4}{4}\right) - \frac{3z}{2} + \frac{9}{2}z$$

$$= \frac{27}{12} - \frac{108}{12} + 3z = 3z - \frac{81}{12}$$

$$\int_0^3 3z - \frac{81}{12} \, dz = \frac{3z^2}{2} - \frac{81}{12}z \Big]_0^3$$

$$= \left(\frac{6}{6}\right)\frac{27}{2} - \frac{243}{12} = \frac{162}{12} - \frac{243}{12}$$

$$= \frac{-81}{12} = \boxed{\frac{-27}{4}}$$

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Section 15.3 - #7, 9, 11, 13, 15, 17:

(7)
$$\int_0^c \int_0^b \int_0^a (x+z)^3 \, dx\, dy\, dz$$

Inner: $\int_0^a (x+z)^3 \, dx$ $\quad u = x+z$, $du = dx$

$$\int_0^a u^3 \, du = \frac{u^4}{4}\Big]_0^a = \frac{(x+z)^4}{4}\Big]_0^a$$

$$\frac{(a+z)^4}{4} - \frac{(z)^4}{4}$$

Middle: $\int_0^b \frac{(a+z)^4}{4} - \frac{(z)^4}{4} \, dy$

$$\frac{y(a+z)^4}{4}\Big]_0^b - \frac{y(z)^4}{4}\Big]_0^b$$

$$\frac{b(a+z)^4}{4} - \frac{b(z)^4}{4}$$

Outer: $\int_0^c \frac{b(a+z)^4}{4} - \frac{b(z)^4}{4} \, dz$

$=$ $\quad u = a+z$, $du = dz$

$$= \frac{b}{4}\left(\frac{(a+z)^5}{5}\right)\Big]_0^c - \frac{b}{4}\left(\frac{z^5}{5}\right]_0^c$$

$$= \frac{b}{4}\left(\frac{(a+c)^5}{5} - \frac{(a)^5}{5}\right) - \frac{b}{4}\left(\frac{c^5}{5}\right)$$

$$= \frac{b(a+c)^5}{20} - \frac{b(a)^5}{20} - \frac{b(c^5)}{20} = \boxed{\frac{(a+c)^5 b - a^5 b - c^5 b}{20}}$$

(9) $F(x,y,z) = x+y$

$$\int_0^1\int_0^x\int_y^x (x+y)\,dz\,dy\,dx$$

Inner:

$$\int_y^x (x+y)\,dz = xz+yz\Big]_y^x = xz\Big]_y^x + yz\Big]_y^x$$
$$= x^2 - xy + xy - y^2$$

Middle:

$$\int_0^x x^2 - xy + xy - y^2\,dy = x^2y - \frac{xy^2}{2} + \frac{xy^2}{2} - \frac{y^3}{3}\Big]_0^x$$
$$= x^3 - \frac{x^3}{2} + \frac{x^3}{2} - \frac{x^3}{3}$$
$$= \left(\frac{3}{3}\right)\frac{x^3}{1} - \frac{x^3}{3} = \frac{2x^3}{3}$$

+1

Outer:

$$\int_0^1 \frac{2x^3}{3}\,dx = \frac{2x^4}{12}\Big]_0^1 = \frac{x^4}{6}\Big]_0^1$$
$$= \boxed{\frac{1}{6}}$$

(11)

$$\int_0^1\int_0^1\int_0^{\sqrt{1-x^2}} xyz\,dy\,dx\,dz$$

Inner:

$$xz\int_0^{\sqrt{1-x^2}} y\,dy = \frac{y^2}{2}\Big]_0^{\sqrt{1-x^2}} = xz\left(\frac{1-x^2}{2}\right)$$

$$\int_0^1 \frac{xz - x^3z}{2}\,dx = \frac{x^2z}{4}\Big]_0^1 - \frac{x^4z}{8}\Big]_0^1$$
$$= \left(\frac{2}{2}\right)\frac{z}{4} - \frac{z}{8} = \frac{z}{8}$$

$$\int_0^1 \frac{z}{8}\,dz = \frac{z^2}{16}\Big]_0^1 = \boxed{\frac{1}{16}}$$

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Section 15.3 - #13, 15, 17:

\#(13) $f(x,y,z) = e^z$ $\quad W = x+y+z \leq 1,\ x \geq 0,\ y \geq 0,\ z \geq 0$

$z = 1 - x - y$

if $z = 0$: $x + y = 1$ → graph look at x and y bounds

$x = 1 - y \quad y = 1 - x$

↳ However you look at it either y goes from 0 to 1-x or x goes from 0 to 1-y

Ask in Recitation

Inner:

$$\int_0^{1-x-y} e^z\,dz = e^{1-x-y} - 1$$

Middle:

$$\int_0^{1-y} \left(e^{1-x-y} - 1\right) dx \qquad u = 1-x-y,\quad \frac{du}{-1} = \frac{-dx}{-1}$$

$$= -\left(e^u\right]_0^{1-y} - x\Big]_0^{1-y}$$

$$= -\left(e^{1-(1-y)-y} - e^{1-y}\right) - (1-y)$$

$$= -e^{1-1+y-y} - e^{1-y} - 1 + y = -e^0 - e^{1-y} - 1 + y$$

$$= -1 - e^{1-y} - 1 + y$$

$$= -e^{1-y} - 2 + y$$

$$\int_0^1 \left(-e^{1-y} - 2 + y\right) dy = -e^{1-y}\Big]_0^1 - 2y\Big]_0^1 + \frac{y^2}{2}\Big]_0^1$$

$$= -(e^0 - e^1) - (2(1) - 2(0)) + \left(\frac{1}{2}\right)$$

$$= -1 + e^1 - \frac{2}{1}\left(\frac{2}{2}\right) + \frac{1}{2}\left(\frac{1}{1}\right)$$

$$= \left(\frac{2}{2}\right)\frac{-1}{1} + e - \frac{3}{2} = \boxed{e - \frac{5}{2}}$$

(15) z goes from 0 to $\sqrt{9-x^2-y^2}$
y goes from 0 to $x \to$ Vertical Bars from x axis to $y=x \to$ line
x goes from 0 to 1:

$$\int_0^1 \int_0^x \int_0^{\sqrt{9-x^2-y^2}} z \, dz\,dy\,dx$$

Inner:

$$\int_0^{\sqrt{9-x^2-y^2}} z\,dz = \frac{z^2}{2}\Big]_0^{\sqrt{9-x^2-y^2}} = \frac{9-x^2-y^2}{2}$$

Middle:

$$\frac{1}{2}\int_0^x 9-x^2-y^2\,dy = \frac{1}{2}\left(9y - yx^2 - \frac{y^3}{3}\right]_0^x$$

$$= \frac{9x}{2} - \left(\frac{3}{3}\right)\frac{x^3}{2} - \frac{x^3}{6} = \frac{9x}{2} - \frac{4x^3}{6}$$

Outer:

$$= \int_0^1 \frac{9x}{2} - \frac{2x^3}{3}\,dx = \frac{9}{2}\int_0^1 x\,dx - \frac{2}{3}\int_0^1 x^3\,dx$$

$$= \frac{9}{2}\left(\frac{x^2}{2}\right)_0^1 - \frac{2}{3}\left(\frac{x^4}{4}\right]_0^1$$

$$= \left(\frac{3}{3}\right)\frac{9}{4} - \frac{2}{12} = \frac{27}{12} - \frac{2}{12} = \boxed{\frac{25}{12}}$$

(17) Bounds $0 \le z \le 8-2x^2-y^2 \to x^2+y^2=4$
$0 \le y \le 2 \quad 0 \le x \le \sqrt{4-y^2}$

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{8-2x^2-y^2} x \, dz\,dx\,dy$$

$$\int_0^2 \int_0^{\sqrt{4-y^2}} x(8-2x^2-y^2)\,dx\,dy$$

$u = 8-2x^2-y^2$
$\frac{du}{-4} = \frac{-4x}{-4}\,dx$

$$\int_0^2 \frac{1}{2}y^4 - 4y^2 + 8\,dy = \boxed{\frac{128}{15}}$$

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Section 15.4: #1, 5, 9, 19, 27, 31, 47, 51:

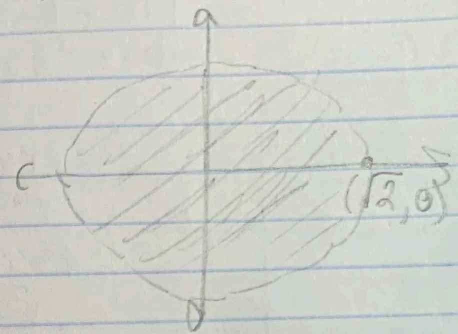
(1) $F(x,y) = \sqrt{x^2+y^2}$, $x^2+y^2 \le 2$

$\sqrt{x^2+y^2} = r$

Radius $= \sqrt{2}$

$0 \le r \le \sqrt{2}$

$0 \le \theta \le 2\pi$.



$$\int_0^{2\pi}\int_0^{\sqrt{2}} r \, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^{\sqrt{2}} r^2 \, dr\, d\theta = \frac{r^3}{3}\Big]_0^{\sqrt{2}} = \frac{(\sqrt{2})^3}{3}$$

$$\int_0^{2\pi} \frac{(\sqrt{2})^3}{3} d\theta = \frac{(\sqrt{2})^3}{3}\theta \Big]_0^{2\pi} = \boxed{\frac{4\sqrt{2}\pi}{3}}$$

(5) $f(x,y) = y(x^2+y^2)^{-1} = \frac{y}{x^2+y^2} = \frac{r\sin\theta}{r^2}$ $\quad y \ge \frac{1}{2}$ $\quad x^2+y^2 \le 1$

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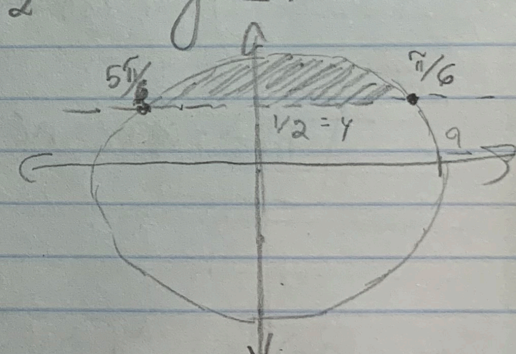
$$\int_{\pi/6}^{5\pi/6}\int_{\frac{\csc\theta}{2}}^{1} \frac{r\sin\theta}{r^2} \, r\, dr\, d\theta$$

Maple

$$= \boxed{\sqrt{3} - \frac{\pi}{3}}$$



$\le r \le 1$

goes until 1

$\sqrt{1-x^2} = y$

⑨ $\displaystyle\int_{0=x}^{1/2=x} \int_{\sqrt{3}x=y}^{\sqrt{1-x^2}=y} x \, dy \, dx$

$x \to r\cos\theta$

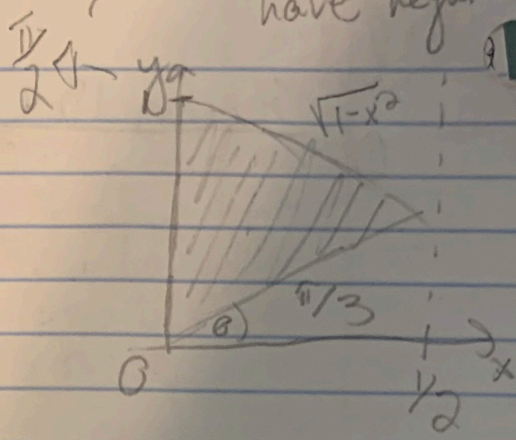
$^2(r\sin\theta = \sqrt{1-r^2\cos^2\theta})$

$r^2\sin^2\theta + r^2\cos^2\theta = 1$

$\sqrt{3}\, r\cos\theta = r\sin\theta$

$\sqrt{3} = \tan\theta \quad \theta = \frac{\pi}{3}$

→ until $\pi/2$ b/c can't have negative x



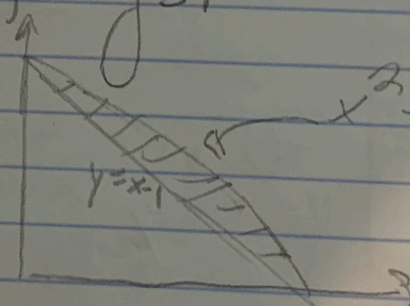
$0 \le r \le 1 \qquad \pi/3 \le \theta \le \pi/2$

$$\int_{\pi/3}^{\pi/2} \int_0^1 r\cos\theta \, r\,dr\,d\theta = \frac{1}{3} - \frac{\sqrt{3}}{6} = .045$$

⑲ $F(x,y) = x-y \; ; x^2+y^2 \le 1, \; x+y \ge 1$

$y = x-1$

$$\int_0^{\pi/2} \int_{\frac{1}{\cos\theta+\sin\theta}}^{1} (r\cos\theta - r\sin\theta)\, r\,dr\, d\theta = 0$$



$r^2\cos^2\theta + r^2\sin^2\theta = 1$

$r = \frac{1}{\cos\theta + \sin\theta}$

$1\sin^2\theta$

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Section 15.4 - # 27, 31, 47, 51:

(27) $f(x,y,z) = x^2+y^2 \quad x^2+y^2 \le 9 \quad 0 \le z \le 5$

$\downarrow$

r^2

$0 \le r \le 3$

$0 \le \theta \le 2\pi$

$$\int_0^5 \int_0^{2\pi} \int_0^3 r^2 \, r dr d\theta dz =$$

Inner:

$$\int_0^3 r^2 r dr = \frac{r^4}{4}\Big]_0^3 = \frac{81}{4}$$

$$\frac{81}{4}\int_0^5 \int_0^{2\pi} d\theta dz = \frac{81}{4}\cdot 2\pi \int_0^5 dz = \frac{81}{4}\cdot 2\pi \cdot 5 = \boxed{\frac{405\pi}{2}}$$

(31) $f(x,y,z) = z; \; x^2+y^2 \le z \le 9$

$\underbrace{r^2 \le z \le 9}_{z \text{ integral}}$ $\quad r^2 \le 9$

$0 \le r \le 3$

$0 \le \theta \le 2\pi$

maple $\rightarrow$ $$\int_0^3 \int_0^{2\pi} \int_{r^2}^9 z \, dz \, d\theta \, r dr = \boxed{243\pi}$$

(47) $f(x,y,z) = x^2+y^2 \quad \rho \le 1$

$= (\rho\cos\theta \sin\phi)^2 + (\rho\sin\theta\sin\phi)^2 \quad \rho^2(\cos^2\theta+\sin^2\theta)\sin^2\phi$

$= \rho^2 \sin^2\phi$

$0 \le \theta \le 2\pi; \; 0 \le \phi \le \pi, \; 0 \le \rho \le 1$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^1 (\rho^2\sin^2\phi)\,\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \boxed{\frac{8\pi}{15}}$$

(5) $f(x,y,z) = z = \rho\cos\phi$ $\quad 0 \le \theta \le \pi/3 \quad 0 \le \phi \le \pi/2$

$1 \le \rho \le 2$

$$\int_0^{\pi/3} \int_0^{\pi/2} \int_1^2 (\rho\cos\phi)\,\rho^2 \sin\phi \; d\rho\, d\phi\, d\theta$$

$$= \boxed{\frac{5\pi}{8}}$$