15.3 Homework

(3) $f(x,y,z) = xe^{y-2z}$, $x:[0,2]$, $y:[0,1]$, $z:[0,1]$

$$\boxed{\int_0^2\int_0^1\int_0^1 xe^{y-2z}\,dz\,dy\,dx = \frac{(e-1)^2(1+e)}{e^2}}$$

(5) $f(x,y,z) = (x-y)(y-z)$; $[0,1]\times[0,3]\times[0,3]$

$$\boxed{\int_0^1\int_0^3\int_0^3 (x-y)(y-z)\,dz\,dy\,dx = -\frac{27}{4}}$$

(7) $f(x,y,z) = (x+z)^3$; $[0,a]\times[0,b]\times[0,c]$

$$\int_0^a\int_0^b\int_0^c (x+z)^3\,dz\,dy\,dx$$

$$\int_0^c (x+z)^3\,dz = \left[\frac{z^4}{4} + xz^3 + \frac{3x^2z^2}{2} + x^3z\right]_0^c$$

$$\int_0^b \left(\frac{c^4}{4} + c^3x + \frac{3c^2x^2}{2} + cx^3\right)dy = \left[\frac{c^4}{4}y + c^3xy + \frac{3c^2x^2}{2}y + cx^3y\right]_0^b$$

$$\int_0^a \left(\frac{bc^4}{4} + bc^3x + \frac{3bc^2x^2}{2} + bcx^3\right)dx = \left[\frac{bc^4}{4}x + \frac{bc^3x^2}{2} + \frac{3bc^2x^3}{6} + \frac{bcx^4}{4}\right]_0^a$$

$$\boxed{\frac{abc^4}{4} + \frac{a^2bc^3}{2} + \frac{a^3bc^2}{2} + \frac{a^4bc}{4}}$$

(11) $f(x,y,z) = xyz$ W: $z:[0,1]$, $y:[0,\sqrt{1-x^2}]$, $x:[0,1]$

$$\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^1 xyz\,dz\,dy\,dx$$

$$\int_0^1 xyz\,dz = \left[\frac{xyz^2}{2}\right]_0^1 = \frac{xy}{2}$$

$$\int_0^{\sqrt{1-x^2}} \frac{xy}{2}\,dy = \left[\frac{xy^2}{4}\right]_0^{\sqrt{1-x^2}} = \frac{x-x^3}{4}$$

$$\int_0^1 \left(\frac{x-x^3}{4}\right)dx = \left[\frac{x^2}{8} - \frac{x^4}{16}\right]_0^1 = \frac{2}{16} - \frac{1}{16}$$

$$\boxed{\iiint_W xyz\,dV = \frac{1}{16}}$$

(13) $f(x,y,z) = e^z$; W: $x+y+z \le 1$, $x \ge 0$, $y \ge 0$, $z \ge 0$

$$\iint_D \left(\int_0^{1-x-y} e^z\,dz\right)dA = \iint_0 \left(e^z\Big|_0^{1-x-y}\right)dA$$

$$\int_0^1 \left(\int_0^{1-y} (e^{1-x-y} - 1)\,dx\right)dy$$

$$\int_0^1 \left(e^{1-y} + y - 2\right)dy = -e^{1-y} + \frac{y^2}{2} - 2y\Big|_0^1$$

$$\boxed{\iiint_W e^z\,dA = e - \frac{5}{2}}$$

(15) $f(x,y,z) = z$, $x = 1$, $y = 0$, $x = y$

$$\iiint_W z\,dV = \iint_D \left(\int_0^{\sqrt{9-x^2-y^2}} z\,dz\right)dA$$

$$\int_0^1 \int_0^x \frac{9-x^2-y^2}{2}\,dy\,dx = \int_0^1 \left(\frac{9y - x^2y - \frac{y^3}{3}}{2}\right)\Big|_0^x dx$$

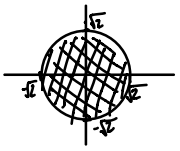
$$\int_0^1 \left(\frac{9x}{2} - \frac{2x^3}{3} \right) dx = \frac{25}{12}$$

$$\boxed{\iiint_W z \, dV = \frac{25}{12}}$$

(17) $f(x,y,z) = x$; $x \geq 0, y \geq 0, z \geq 0$ above $z = y^2$ below $z = 8 - 2x^2 - y^2$

$$\boxed{\iiint_W x \, dV = \frac{128}{15}}$$

① $f(x,y) = \sqrt{x^2+y^2}$, $x^2 + y^2 \le 2$

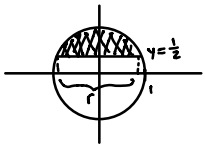


$\iint_0 \sqrt{x^2+y^2}\,dA$

$f(x,y) \to g(r,\theta)$

$$\int_0^{2\pi}\int_0^{\sqrt{2}} r \cdot r\,dr\,d\theta = \frac{4\sqrt{2}}{3}\pi$$

⑤ $f(x,y) = y(x^2+y^2)^{-1}$; $y \ge \frac{1}{2}$, $x^2+y^2 \le 1$



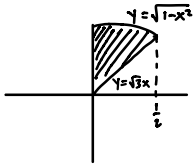
$\iint_D y(x^2+y^2)^{-1}\,dA$

$\theta = \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

$r = \left[\frac{\csc\theta}{2}, 1\right]$

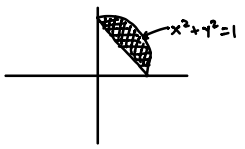
$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\int_1^{\frac{\csc\theta}{2}} \frac{\sin\theta}{r}\,r\,dr\,d\theta = \sqrt{3} - \frac{\pi}{3}$$

⑨ $\int_0^{1/2}\int_{\sqrt{3}x}^{\sqrt{1-x^2}} x\,dy\,dx$



$$\int_0^{1/2}\int_{\sqrt{3}x}^{\sqrt{1-x^2}} x\,dy\,dx = \frac{1}{3}\left(1 - \frac{\sqrt{3}}{2}\right)$$

⑲ $f(x,y) = x - y$; $x^2+y^2 \le 1$; $x + y \ge 1$



$r = \left[\frac{1}{\cos\theta+\sin\theta}, 1\right]$

$\theta = \left[0, \frac{\pi}{2}\right]$

$$\int_0^{\frac{\pi}{2}}\int_{\frac{1}{\cos\theta+\sin\theta}}^{1} r^2(\cos\theta - \sin\theta)\,dr\,d\theta = 0$$

(27) $f(x,y,z) = x^2+y^2;\ x^2+y^2 \le 9,\ 0 \le z \le 5$

$$\int_0^5 \int_0^{2\pi} \int_0^3 r^3 \, dr\, d\theta\, dz = \frac{405\pi}{2}$$

(31) $f(x,y,z) = z;\ x^2+y^2 \le z \le 9$

$$\int_0^3 \int_0^{2\pi} \int_{r^2}^9 z\, r \, dz\, dr\, d\theta = 243\pi$$

(47) $f(x,y,z) = x^2+y^2;\ \rho \le 1$

$$\int_0^{2\pi} \int_0^{\pi} \int_0^1 (e^2 \sin^2\theta) e^2 \sin\theta \, de\, d\theta = \frac{8\pi}{15}$$

(51) $f(x,y,z) = z;\ 0 \le \theta \le \frac{\pi}{3};\ 0 \le \phi \le \frac{\pi}{2};\ 1 \le \rho \le 2$

$$\int_0^{\frac{\pi}{3}} \int_0^{\frac{\pi}{2}} \int_1^2 (e\cos\theta)\, e^2 \sin\phi \, de\, d\phi\, d\theta = \frac{5\pi}{8}$$