15.3: 3, 5, 7 9 11 13 15 17
15.4: 1 5 9 19 27 31 47 51

Chap 15 HW

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15.3

3) $\int_0^1\int_0^1\int_0^2 xe^{-y-2z}\,dx\,dy\,dz =$

$2\int_0^1\int_0^1 e^{-y-2z}\,dy\,dz = 2\int_0^1\int_0^1 e^{-y}e^{-2z}\,dy\,dz$

$= -2(e^{-1}-1)\int_0^1 e^{-2z}\,dz = \boxed{+1(e^{-1}-1)(e^{-2}-1)}$

5) $\int_0^3\int_0^3\int_0^1 (x-y)(y-z)\,dx\,dy\,dz =$

$\int_0^3\int_0^3 (y-z)\left(\frac{x^2}{2}-yx\right)\Big|_0^1 dy\,dz = \int_0^3\int_0^3 (y-z)\left(\frac{1}{2}-y\right)dy$

$= \int_0^3\int_0^3 -y^2 + y\left(z+\frac{1}{2}\right) - \frac{z}{2}$

7) $\int_0^c\int_0^b\int_0^a (x+z)^3\,dx\,dy\,dz = \int_0^c\int_0^b \frac{(a+z)^4}{4} - \frac{z^4}{4}\,dy\,dz$

$= \int_0^c b\left(\frac{(a+z)^4}{4} - \frac{z^4}{4}\right)dz = b\left(\frac{(a+z)^5}{20} - \frac{z^5}{20}\right)\Big|_0^c$

$= \boxed{b\left(\frac{(a+c)^5 - c^5 - a^5}{20}\right)}$

15.3 Cont

9) $$\int_0^1\int_0^x\int_y^x x+y\,dz\,dy\,dx = \int_0^1\int_0^x (x+y)(x-y)\,dy\,dx$$

$$= \int_0^1\int_0^x x^2-y^2\,dy\,dx = \int_0^1\left(x^2y-\frac{y^3}{3}\right)\Big|_0^x dx$$

$$= \int_0^1 x^3-\frac{x^3}{3}\,dx = \int_0^1 \frac{2x^3}{3}\,dx = \frac{x^4}{6} = \boxed{\frac{1}{6}}$$

11) $$\int_0^1\int_0^1\int_0^{\sqrt{1-x^2}} xyz\,dy\,dz\,dx = \int_0^1\int_0^1 xz\frac{(1-x^2)}{2}\,dz\,dx$$

$$= \int_0^1\frac{1}{4}x(1-x^2)\,dx = \frac{1}{4}\left(\frac{x^2}{2}-\frac{x^4}{4}\right)\Big|_0^1 = \boxed{\frac{1}{16}}$$

13) $$\int_0^{\frac{1}{2}}\int_0^{\frac{1}{2}}\int_0^{1-x-y} e^z\,dz\,dy\,dx \quad W: x+y+z\le 1; \quad \begin{matrix} x\ge 0 \\ y\ge 0 \\ z\ge 0\end{matrix}$$

$$= \int_0^{\frac{1}{2}}\int_0^{\frac{1}{2}} e\cdot e^{-x}\cdot e^{-y}\,dy\,dx = \int_0^{\frac{1}{2}} -e(e^{-\frac{1}{2}}-1)e^{-y}$$

$$= \int_0^{1/2} -\sqrt{e}\,e^{-y}\,dy = \boxed{+e(e^{-1/2}-1)(e^{-1/2}-1)}$$

15.3 Cont

15) $\iiint_W z\,dV \quad W = \{ x^2 + y^2 + z^2 = 9$

$0 \le z \le \sqrt{9 - x^2 - y^2}$

$y \le x \le 1$

$0 \le y \le 1 \}$

$$\int_0^1 \int_y^1 \int_0^{\sqrt{9-x^2-y^2}} z\,dz\,dx\,dy$$

$$= \frac{1}{2}\int_0^1 \int_y^1 9 - x^2 - y^2\,dx\,dy = \frac{1}{2}\int_0^1 \left(9x - xy^2 - \frac{x^3}{3}\right)\Big|_y^1 dy$$

$$= \frac{1}{2}\int_0^1 \left(9 - y^2 - \frac{1}{3}\right) - \left(9y - y^3 - \frac{y^3}{3}\right) dy =$$

$$= \frac{1}{2}\int_0^1 9 - \frac{1}{3} - y^2 - 9y + \frac{4y^3}{3}\,dy = \frac{1}{2}\int_0^1 \frac{4y^3}{3} - y^2 - 9y + \frac{26}{3}\,dy$$

$$= \frac{1}{2}\left(\frac{y^4}{3} - \frac{y^3}{3} - \frac{9y^2}{2} + \frac{26y}{3}\right)\Big|_0^1 = \frac{1}{2}\left(0 - \frac{9}{2} + \frac{26}{3}\right)$$

$$= \boxed{\frac{25}{12}}$$

17) $\iiint_W x\,dV \quad W = \{ (x, y, z)$

$y^2 \le z \le 8 - 2x^2 - y^2$

$0 \le y \le \sqrt{4 - x^2}$

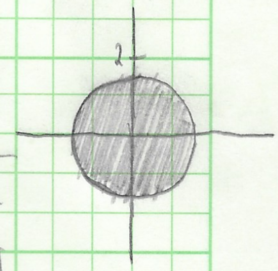
$0 \le x \le 2$

$$= \int_0^2 \int_0^{\sqrt{4-x^2}} \int_{y^2}^{8-2x^2-y^2} x\,dz\,dy\,dx = \boxed{\frac{128}{15}}$$

15.4 HW

0:

1) $f(x, y) = \sqrt{x^2+y^2}$; $f(r, \theta) = r$ $r^2 \le 2$



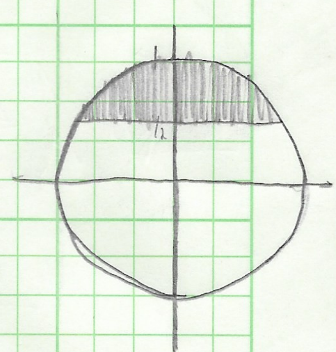
$$\int_0^{2\pi}\int_{-\sqrt{2}}^{\sqrt{2}} r \cdot r \, dr \, d\theta = \int_0^{2\pi}\left(\frac{(2)^{3/2}}{3} - \frac{-(2)^{3/2}}{3}\right)$$

$-\sqrt{2} \le r \le \sqrt{2}$

$0 \le \theta \le 2\pi$

$$= \frac{(2)^{5/2}}{3} = \boxed{\frac{\sqrt{32}}{3}} = \boxed{\frac{4\sqrt{2}}{3}}$$

5) $f(x, y) = y(x^2+y^2)^{-1}$ $f(r, \theta) = \frac{r\sin\theta}{r^2}$



$x^2 + y^2 = r^2 \le 1$ $y = r\sin\theta \ge \frac{1}{2}$

$-1 \le r \le 1$ $r \ge \frac{1}{2\sin\theta}$

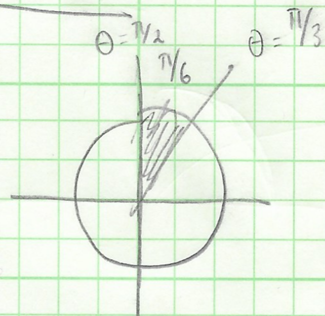
$$= \int_{\pi/6}^{5\pi/6}\int_{\frac{1}{2\sin\theta}}^{1} \sin\theta \, dr \, d\theta = \int_{\pi/6}^{5\pi/6} \sin\theta\left(1 - \frac{1}{2\sin\theta}\right) d\theta$$

$$= \int_{\pi/6}^{\frac{5\pi}{6}} \sin\theta - \frac{1}{2} \, d\theta = \left(-\cos\theta - \frac{\theta}{2}\right)\Big|_{\pi/6}^{5\pi/6}$$

$$= \left(\frac{\sqrt{3}}{2} - \frac{5\pi}{12}\right) - \left(-\frac{\sqrt{3}}{2} - \frac{\pi}{12}\right) = \boxed{\sqrt{3} - \frac{\pi}{3}}$$

15.4 Cont

9) $\displaystyle\int_0^{1/2}\int_{(\sqrt{3})x}^{\sqrt{1-x^2}} x\,dy\,dx$



$\sqrt{3}x = \sqrt{1-x^2}$ $\quad 3x^2 = 1-x^2$

$4x^2 = 1$

$x = \pm\frac{1}{2}$

$$= \int_{\pi/3}^{\pi/2}\int_0^1 r\cos\theta \cdot r\,dr\,d\theta = \int_{\pi/3}^{\pi/2}\left(\frac{1}{3}\right)\sin\theta\,d\theta = \frac{1}{3}\left(-0-\left(-\frac{1}{2}\right)\right)$$

$$= \boxed{\frac{2-\sqrt{3}}{6}}$$

19) $f(x, y) = x - y$ $\quad x^2+y^2 = r^2 \leq 1$ $\quad x+y \geq 1$

$r = \pm 1$ $\quad r(\sin\theta+\cos\theta) \geq 1$

$$\int_0^{\pi/2}\int_{\frac{1}{(\sin\theta+\cos\theta)}}^{1} r(\sin\theta-\cos\theta)\,r\,dr\,d\theta$$

$r \geq \frac{1}{\sin\theta+\cos\theta}$

$D: \{(r, \theta) \mid 0 \leq \theta \leq \frac{\pi}{2};\ \frac{1}{\sin\theta+\cos\theta} \leq r \leq 1\}$

$= \boxed{0}$ found Using MAPLE, likely wrong

15.4 Cont

27) $f(x,y,z) = x^2 + y^2 \quad x^2 + y^2 \le 9 \quad 0 \le z \le 5$

$r^2 \le 9$

$$\int_0^5 \int_0^{2\pi} \int_0^3 r^2 r \, dr \, d\theta \, dz = \frac{3^4}{4} \cdot 10\pi = \boxed{\frac{405\pi}{2}}$$

31) $$\int_0^{2\pi} \int_0^3 \int_{r^2}^9 z r \, dz \, dr \, d\theta = \int_0^{2\pi} \int_0^3 r\left(\frac{81}{2} - \frac{r^4}{2}\right) dr \, d\theta$$

$$= \int_0^{2\pi} \frac{1}{2}\left(\frac{81 r^2}{2} - \frac{r^5}{2}\right)\Big|_0^3 d\theta = \frac{1}{2}\int_0^{2\pi} 243 \, d\theta = \boxed{243\pi}$$

47) $f(x,y,z) = x^2 + y^2 \quad \rho \le 1$

$x^2 + y^2 = (\rho \sin\phi \cos\theta)^2 + (\rho \sin\phi \sin\theta)^2$

$x^2 + y^2 = \boxed{\rho^2 (\sin\phi)^2} \cancel{((\cos\theta)^2 + (\sin\theta)^2)}$

$f(\rho, \phi, \theta) = \rho^2 \sin^2\phi$

$$= \int_0^{2\pi} \int_0^{2\pi} \int_0^1 \rho^2 \sin^2\phi \, \rho^2 \, d\rho \, d\phi \, d\theta$$

$$= \boxed{\frac{2\pi^2}{5}}$$

15.4 Cont

51) $$\int_0^{\pi/3}\int_0^{\pi/2}\int_1^2 \rho\cos\phi\,\rho^2\,d\rho\,d\phi\,d\theta$$

Done in Maple

$$= \frac{5\,(\sin(\frac{\pi}{2}))\cdot\pi}{2} = \boxed{\frac{5\pi}{2}}$$