Jessenia Bello - Calc homework

15.3

3) $\int_0^2 \int_0^1 \int_0^1 xe^{x-2z}\,dz\,dy\,dx \to \int_0^1 xe^{y-2z}\,dz =$

$xe^y \int_0^1 e^{-2z}\,dz \quad \begin{cases} u=-2z \to [0,-2] \\ du=-2dz \end{cases}$

$\frac{xe^y}{-2}\left[e^u\Big|_0^{-2}\right] = \frac{xe^y}{-2}\left(\frac{1}{e^2}-1\right) \to \int_0^1 \frac{xe^y}{-2}\left(\frac{1}{e^2}-1\right)dy$

$= \frac{-x}{2}\left(\frac{1}{e^2}-1\right)\int_0^1 e^y\,dy = \frac{-x}{2}\left(\frac{1}{e^2}-1\right)(e-1)$

$\int_0^2 -\frac{x}{2}\left(\frac{1}{e^2}-1\right)(e-1)\,dx = -\frac{1}{2}\left(\frac{1}{e^2}-1\right)(e-1)\int_0^2 x\,dx$

$= \boxed{-\tfrac{1}{2}\left(\tfrac{1}{e^2}-1\right)(e-1)[0]} = (1+e^{-2})(e-1)$

5) $f(x,y,z) = (x-y)(y-z) = xy - xz - y^2 + yz$

$[0,1]\times[0,3]\times[0,3]$

$\int_0^1 \int_0^3 \int_0^3 xy - xz - y^2 + yz\,dz\,dx \to \int_0^3 xy - xz - y^2 + yz\,dz$

$= \frac{xz^2}{2} - y^2 z + \frac{yz^2}{2}$

$\frac{27x}{2} - 27 + \frac{9}{2} \to \int_0^1 \left(\frac{9}{2}x - 18\right)dx = \frac{9}{4}x^2 - 18x\Big|_0^1$

$\to \frac{9}{4} - 18 = -15.75$

7) $f(x,y,z) = (x+z)^3 \quad [0,1]\times[0,2]\times[0,3]$

$\int_0^3 \int_0^2 \int_0^1 (x+z)^3\,dx\,dz\,dy \to \int_0^1 \int_0^3 (x+z)^3\,dz\,dx$

$u = x+z \quad du = dx \quad \int_{x=b}^{x=a} u^3\,du$

$\frac{u^4}{4}\Big|_{x=0}^{x=1} \quad \frac{(x+z)^4}{4}\Big|_0^3 = \frac{z^4}{4}$

$\int_0^3 \frac{(a+c)^5}{20} - \frac{c^5}{20}\,dy = \frac{b}{20}\left[a+c^5 - c^5\right]$

$\frac{b}{8} 0 \quad ((9+c)^{2} - c^{2})$

9) $f(x,y,z) = x+y$ w $y \le z \le x$

$\int_0^1 \int_0^x \int_y^x x+y \, dz\, dy\, dx$

$\int (x+y) dz = (x + x \times x - y) = \int_0^x x^2 - y^2 dy = x^2 y - \frac{y^3}{3} \Big|_0^x$

$x^3 - \frac{x^3}{3} \to \int_0^1 \frac{x \; 3x^3}{3} = \frac{x^4}{4} - \frac{x^4}{12} \Big|_0^1 = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$

11) $f(x,y,z) = xyz$ $W = 0 \le z \le 1$ $0 \le \sqrt{1-x^2}$
$0 \le x \le 1$

$\int_0^1 \int_0^1 \int_0^{\sqrt{1-x^2}} xyz$ $W;\ 0 \le z \le 1$ $0 \le y \le \sqrt{1-x^2}$
$0 \le x \le 1$

$\frac{z}{x} \left(\frac{x^2}{2} - \frac{x^4}{4} \Big|_0^1 \right) = \frac{z}{2} \left(\frac{1}{2} - \frac{1}{4} \right) = \int_0^1 \frac{z}{8} \, dz = \frac{1}{8} \frac{z^2}{2} \Big|_0^1$
$= \left(\frac{1}{2}\right)\left(\frac{1}{8}\right) = \frac{1}{16}$

13) $f(x,y,z) = e^z$ w $x+y<1$ $0 \ge x$ $y \ge 0$ $z \ge 0$

$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} e^z \, dz \to \int_0^{1-x-y} e^z dz = e^{1-x-y} - 1$

$u = [1-x, 1-x(1-x) = 0$
$-(1 - e^{1-x}) - (1-x) = e^{1-x} - 1 - 1 + x = x + e^{1-x} - 2$

$\int_0^1 (x + e^{1-x} - 2) dx = e - \frac{5}{2}$

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1) $f(x,y) = \sqrt{x^2+y^2}$ $x^2+y^2 \leq 2$

$x^2+y^2 = 2$ $r = 2$

$\int_0^{2\pi} \int_0^{\sqrt{2}} r \cdot r\, dr\, d\theta \to \int_0^{\sqrt{2}} r^2 dr \to \frac{r^3}{3}\Big|_0^{\sqrt{2}} = \int_0^{2\pi} \frac{2\sqrt{2}}{3} d\theta =$

$\frac{4\sqrt{2}\pi}{3}$

5) $f(x,y) = y(x^2+y^2)^{-1}$ $y \geq 1/2$

$x^2+y^2 \leq 1$

$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \int_{\frac{1}{2}\csc\theta}^{\sqrt{1-x^2}} x\, dy\, dx$

a) $\int_{\pi/3}^{2\pi/3} \int_0^1 r^2 \cos\theta\, dr\, d\theta = \int_0^1 r^2 \cos\theta\, dr$

$= \frac{r^3}{3}\cos\theta\Big|_0^1 = \frac{\cos\theta}{3} \to \int_{\pi/3}^{2\pi/3} \frac{\cos\theta}{3} d\theta$

$1/3 \int_{\pi/3}^{2\pi/3} \cos\theta\, d\theta = 1/3(\sin\theta)\Big|_{\pi/3}^{2\pi/3} = \left(\frac{\sqrt{3}}{2} - 1\right)\left(\frac{1}{3}\right)$

19) $f(x,y) = x-y$ $x^2+y^2 \leq 1$ $x+y \geq 1$

$\int_{\pi/2}^{\pi} \int_0^1 r(\cos\theta - \sin\theta) r\, dr\, d\theta$

$1/3 \int_{\pi/2}^{\pi} \cos\theta - \sin\theta\, d\theta = 1/3(\sin\theta + \cos\theta)\Big|_{\pi/2}^{\pi}$

$= 1/3[[\sin\pi + \cos\pi] - [\sin\frac{\pi}{2} + \cos\frac{\pi}{2}]] = 0$

27) $\int_0^{2\pi} \int_0^5 \int_0^{\sqrt{25-r^2}} r\, dz\, dr\, d\theta = \int_0^{2\pi} \int_0^5 z\Big|_0^{\sqrt{25-r^2}} r\, dr\, d\theta$

$= \int_0^{2\pi} \int_0^5 r\sqrt{25-r^2}\, dr\, d\theta = \frac{125}{4}\pi$

47) $f(x,y,z) = x$ by E

$x = \rho \sin\phi\cos\theta \quad y = \rho\sin\phi\sin\theta \quad z = \rho\cos\phi$

$r = 6\sin\phi \quad 0 \le \rho \le 1 \quad 0 \le \phi \le \pi$

$\rho = 6(\theta, \phi, \rho)$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{1} \rho^4 \sin^3\phi \, d\rho\, d\phi\, d\theta = \frac{8\pi}{15} \text{ mass}$$

51) $\int_0^{\pi/3}\int_0^{\pi/4}\int_1^2 r^3\cos\theta\sin\theta \, dr\, d\theta\, d\theta = \frac{5}{8}$ mass