15.3

3.

$$\int_0^1\int_0^1\int_0^2 xe^{y-2z}\,dx\,dy\,dz$$

Inner Loop: $\left[\frac{x^2}{2}\cdot e^{y-2z}\right]_0^2$

$= 2e^{y-2z}$

Middle Loop: $\int_0^1 2e^{y-2z}\,dy$

$= -2e^{1-2z} + 2e^{-2z}$

Outer Loop: $2\int_0^1 (e^{-2z} - e^{1-2z})\,dz$

$= [e^{-2z} - e^{1-2z}]_0^1$

$= (e-1) + (e^{-1} - e^{-2})$

$= (e-1)(1-e^{-2})$

5. $\int_0^1\int_0^3\int_0^3 (xy - xz - y^2 + yz)\,dy\,dz\,dx$

Inner Loop: $\left[\frac{xy^2}{2} - xyz - \frac{y^3}{3} + \frac{y^2z}{2}\right]_0^3$

$= \frac{9}{2}x - 3xz + \frac{9}{2}z - 9$

Middle Loop: $\int_0^3 (\frac{9}{2}x + \frac{9}{2}z - 3xz - 9)\,dz$

$= \left[\frac{9}{2}xz + \frac{9}{4}z^2 - \frac{3}{2}xz^2 - 9z\right]_0^3$

$= \frac{27}{2}x - \frac{27}{2}x + \frac{81}{4} - 27$

$= -\frac{27}{4}$

Outer Loop: $\int_0^1 -\frac{27}{4}\,dx$

$= -\frac{27}{4}$

7. $\cancel{\int_0^a\int_0^c\int_0^b}$

$$\int_0^b\int_0^a\int_0^c (x+z)^3\,dz\,dx\,dy$$

Inner Loop: $\int_0^c (x+z)^3\,dz.$

$= \left[\frac{(x+z)^4}{4}\right]_0^c$

$= \frac{(x+c)^4 - x^4}{4}$

Middle Loop: $\int_0^a \frac{(x+c)^4 - x^4}{4}\,dx.$

$= \left[\frac{(x+c)^5 - x^5}{20}\right]_0^a$

$= \frac{(a+c)^5 - a^5 - c^5}{20}$

Outer Loop: $\int_0^b \frac{(a+c)^5 - a^5 - c^5}{20}\,dy$

$= \frac{b[(a+c)^5 - a^5 - c^5]}{20}$

9. $\int_0^1\int_0^x\int_y^x (x+y)\,dz\,dy\,dx$

Inner Loop: $\int_y^x (x+y)\,dz$

$= (x+y)(x-y)$

$= x^2 - y^2$

Middle Loop: $\int_0^x (x^2 - y^2)\,dy$

$= \left[x^2y - \frac{1}{3}y^3\right]_0^x$

$= x^3 - \frac{1}{3}x^3$

$= \frac{2}{3}x^3$

Outer Loop: $\frac{2}{3}\int_0^1 x^3\,dx$

$= \frac{2}{3}\cdot\left[\frac{x^4}{4}\right]_0^1$

$= \frac{2}{3}\times\frac{1}{4}$

$= \frac{1}{6}$

11. $\int_0^1 \int_0^1 \int_0^{\sqrt{1-x^2}} xyz \, dy\,dx\,dz$

Inner Loop: $\int_0^{\sqrt{1-x^2}} xyz \, dy$

$= \left[\frac{xy^2z}{2}\right]_0^{\sqrt{1-x^2}}$

$= \frac{xz - x^3z}{2}$

Middle Loop: $\int_0^1 \frac{xz - x^3z}{2} \, dx$

$= \left[\frac{1}{4}x^2z - \frac{1}{8}x^4z\right]_0^1$

$= \frac{1}{4}z - \frac{1}{8}z$

$= \frac{1}{8}z$

Outer Loop: $\int_0^1 \frac{1}{8}z \, dz$

$= \left[\frac{1}{16}z^2\right]_0^1$

$= \frac{1}{16}$

13. $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} e^z \, dz\,dy\,dx$.

Inner Loop: $\int_0^{1-x-y} e^z \, dz$

$= e^{1-x-y} - 1$

Middle Loop: $\int_0^{1-x} (e^{1-x-y} - 1) \, dy$

$= \left[-e^{1-x-y} - y\right]_0^{1-x}$

$= x - 2 + e^{1-x}$.

Outer Loop: $\int_0^1 (x - 2 + e^{1-x}) \, dx$.

$= \left[\frac{x^2}{2} - 2x - e^{1-x}\right]_0^1$

$= \frac{1}{2} - 2 - 1 + e$

$= e - \frac{5}{2}$.

15. $\int_0^1 \int_0^x \int_0^{\sqrt{9-x^2-y^2}} z \, dz\,dy\,dx$.

Inner Loop: $\left[\frac{z^2}{2}\right]_0^{\sqrt{9-x^2-y^2}}$

$= \frac{9-x^2-y^2}{2}$

Middle Loop: $\int_0^x \frac{9-x^2-y^2}{2} \, dy$

$= \left[\frac{9}{2}y - \frac{1}{2}x^2y - \frac{1}{6}y^3\right]_0^x$

$= \frac{9}{2}x - \frac{1}{2}x^3 - \frac{1}{6}x^3$

$= \frac{9}{2}x - \frac{2}{3}x^3$

Outer Loop: $\int_0^1 \frac{9}{2}x - \frac{2}{3}x^3 \, dx$

$= \left[\frac{9}{4}x^2 - \frac{1}{6}x^4\right]_0^1$

$= \frac{9}{4} - \frac{1}{6}$

$= \frac{25}{12}$

17. $y^2 = 8 - 2x^2 - y^2$.

$x^2 + y^2 = 4$.

$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{y^2}^{8-2x^2-y^2} x \, dz\,dy\,dx$.

Inner Loop: $x \cdot (8 - 2x^2 - y^2 - y^2)$

$= 8x - 2x^3 - 2xy^2$

Middle Loop: $\int_0^{\sqrt{4-x^2}} 8x - 2x^3 - 2xy^2 \, dy$

$= \left[8xy - 2x^3y - \frac{2}{3}xy^3\right]_0^{\sqrt{4-x^2}}$

$\cancel{= 8\sqrt{4-x^2}\,x - 2x^3\sqrt{4-x^2} - \frac{2}{3}x(4-x^2)^3}$

$\frac{4}{3}x(4-x^2)^{\frac{3}{2}}$

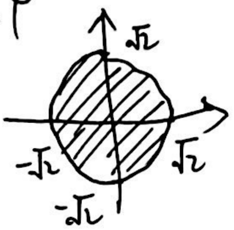
Outer Loop: $\left[-\frac{4(\sqrt{4-x^2})^5}{15}\right]_0^2$

$= \frac{32 \times 4}{15}$

$= \frac{128}{15}$

15.4

1.



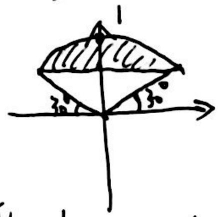
$$\int_0^{\sqrt2}\int_0^{2\pi} r^2\,d\theta\,dr$$

$$=\int_0^{\sqrt2} 2\pi r^2\,dr$$

$$=\left[\frac{2\pi r^3}{3}\right]_0^{\sqrt2}$$

$$=\frac{4\sqrt2}{3}\pi$$

5.



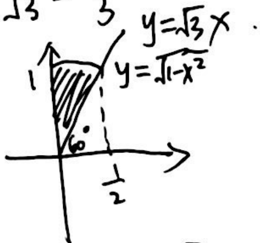
$$\int_{\frac16\pi}^{\frac56\pi}\int_0^1 \sin\theta\,dr\,d\theta-\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}}\int_{\frac{\sqrt3}{3}x}^{\frac12}\frac{y}{x^2+y^2}\,dy\,dx$$

$$=\left[-\cos\theta\right]_{\frac16\pi}^{\frac56\pi}-\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}}\cdot\frac12\left[\ln(x^2+y^2)\right]_{\frac{\sqrt3}{3}x}^{\frac12}dx$$

$$\cancel{=\frac{1}{\sqrt3}-\frac12\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}}\ln(x+\tfrac14)-\ln(\tfrac{4x^2}{3})\,dx}$$

$$=\sqrt3-\frac{\pi}{3}$$

9.



$\cancel{\sqrt3 x=\sqrt{1-x^2}}$

$\cancel{3x^2=1-x^2}$

$\cancel{x=\frac12}$

$\cancel{y=\frac{\sqrt3}{2}}$

$\cancel{\theta=60°}$

$$\int_{\sqrt3 x}^{\sqrt{1-x^2}} x\,dy$$

$$=x\sqrt{1-x^2}-\sqrt3 x^2$$

$$\int_0^{\frac12} x\sqrt{1-x^2}-\sqrt3 x^2\,dx$$

$$=\left[-\frac{(1-x^2)\sqrt{1-x^2}+\sqrt3 x^3}{3}\right]_0^{\frac12}$$

$$=\frac{2-\sqrt3}{6}$$

19. $$\int_0^{\frac12\pi}\int_0^1 r^2(\sin\theta-\cos\theta)\,dr\,d\theta-\int_0^1\int_{1-y}^1(x-y)\,dx\,dy$$

$$=\int_0^{\frac12\pi}\frac13(\sin\theta-\cos\theta)\,d\theta-\int_0^1\left(y-\frac32y^2\right)dy$$

$$=0-0$$

$$=0.$$

27. $$\int_0^5\int_0^{2\pi}\int_0^3 r^3\,dr\,d\theta\,dz$$

$$\cancel{=\int_0^5\int_0^{2\pi}\left[\frac{r^4}{4}\right.}$$

Inner Loop: $\left[\frac{r^4}{4}\right]_0^3=\frac{81}{4}$

Middle Loop: $\frac{81}{4}\times2\pi=\frac{81}{2}\pi$

Outer Loop: $\frac{81}{2}\pi\times5=\frac{405}{2}\pi.$

31. $$\int_0^{2\pi}\int_0^3\int_{r^2}^9 rz\,dz\,dr\,d\theta.$$

Inner Loop: $\left[\frac{rz^2}{2}\right]_{r^2}^9=\frac{81}{2}r-\frac12r^5.$

Middle Loop: $\left[\frac{81}{4}r^2-\frac{1}{12}r^6\right]_0^3=\frac{243}{2}$

Outer Loop: $\frac{243}{2}\times2\pi=243\pi$

47. $$\int_0^{2\pi}\int_0^{2\pi}\int_0^1\rho^4\sin^3\phi\,d\rho\,d\phi\,d\theta$$

Inner Loop: $\left[\frac{\rho^5}{5}\sin^3\phi\right]_0^1=\frac15\sin^3\phi$

Middle Loop: $\left[\frac{\cos^3\phi}{15}-\frac{\cos\phi}{5}\right]_0^{2\pi}=0.$

Outer Loop: $0.$

51. $$\int_0^{\frac{\pi}{3}}\int_0^{\frac{\pi}{2}}\int_1^2\rho^3\sin\phi\cos\phi\,d\rho\,d\phi\,d\theta$$

Inner Loop: $\left[\frac{\rho^4}{4}\sin\phi\cos\phi\right]_1^2=\frac{15}{4}\sin\phi\cos\phi$

Middle Loop: $\left[-\frac{15}{16}\cos2\phi\right]_0^{\frac{\pi}{2}}=\frac{15}{8}$

Outer Loop: $\frac{15}{8}\times\frac{\pi}{3}=\frac58\pi.$