homework 8

15.3

3.) $f(x,y,z) = xe^{y-2z}$

$\int_0^2 \int_0^1 \int_0^1 xe^{y-2z}\, dz\,dy\,dx$

$\int_0^1 xe^{y-2z}\, dz \rightarrow -z(z \cdot xe^{y}) \Big|_0^1 = xe^{y-1}$

$\int_0^1 xe^{y-1}\, dy \rightarrow xe^{y-y} \Big|_0^1 = (e-1)x-1$

$\int_0^2 (e-1)x-1\, dx \rightarrow \frac{x((e-1)x-2)}{2} \Big|_0^2 = \boxed{2(e-2)}$

5.) $f(x,y,z) = (x-y)(y-z)$

$\int_0^1 \int_0^3 \int_0^3 (x-y)(y-z)\, dz\,dy\,dx$

$\int_0^3 (x-y)(y-z)\, dz \rightarrow \frac{(y-z)z(z-2y)}{2} \Big|_0^3$

$\int_0^3 \frac{-6y^2-6xy-9y+9x}{2}\, dy \rightarrow \frac{-y(y(4y+6x+9)-18x)}{4} \Big|_0^3$

$= \boxed{\frac{27}{4}}$ no $\int_0^1 f(x)\, dx$

7.) $f(x,y,z) = (x+z)^3$

$\int_0^a \int_0^b \int_0^c (x+z)^3\, dz\,dy\,dx$

$\int_0^c (x+z)^3\, dz \rightarrow \frac{(x+z)^4}{4} \Big|_0^c =$

$\int_0^b \frac{c(4x^3+6cx^2+4c^2x+c^3)}{4}\, dy = \frac{c(2x+c)(2x^2+2cx+c^2)y}{4} \Big|_0^b$

$\int_0^a b\left(\frac{1}{2}x^3 + \frac{3c^2x^2}{2} + c^3x + \frac{c^4}{4}\right) dx = \frac{bcx(x+c)(x^2+cx+c^2)}{4} \Big|_0^a$

$\boxed{\frac{abc(a^3+2a^2c+2ac^2+c^3)}{4}}$

11.) $f(x,y,z) = xyz$

$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^1 \rightarrow \int_0^1 \int_0^1 \int_0^{\sqrt{1-x^2}} xyz \; dy\,dx\,dz$

$\int_0^{\sqrt{1-x^2}} xyz \, dy \rightarrow \frac{xzy^2}{2} \Big|_0^{\sqrt{1-x^2}} =$

$\int_0^1 \frac{-(x^3-x)}{2} z \, dx \rightarrow -\frac{2x^2(x^2-2}{8} \Big|_0^1 = \frac{2}{8}$

$\int_0^1 \frac{2}{8} \, dz = \frac{z^2}{16} \Big|_0^1 = \boxed{\frac{1}{10}}$

15.) $f(x,y,z) = x^2+y^2+x^2=9$

$\int_0^1 \int_1^4 \int_0^4 x^2+y^2+x^2 \rightarrow \int_0^1 \int_0^1 \int_0^4 x^2+y^2+z^2 \, dx\,dz\,dy$

$\int_1^4 x^2+y^2+x^2 \, dx \Rightarrow \frac{x^3}{3} + (z^2+y^2)x \Big|_0^4 = \frac{3yz^2+4y^3}{3}$

$\int_0^9 \frac{3yz^2+4y^3}{3} \, dy\,dz = \frac{yz^3+4y^3z}{3} \Big|_0^9 =$

$\int_0^1 12y^3+243y \, dy \rightarrow \frac{3y^4+243y^2}{2} \Big|_0^1 = \boxed{\frac{249}{2}}$

17.) $f(x,y,z) = x$

$z = 8-2x^2-y^2$

$x^2+y^2 = 4$

$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{y^2}^{8-2x^2-y^2} x \, dz\,dx\,dy$

$\int_{y^2}^{8-2x^2-y^2} x \, dz \rightarrow xz \Big|_{y^2}^{8-2x^2-y^2} = x(8-2x^2-y^2) = \frac{-(2x^2-2y^2+8)}{8} \Big|_0^{\sqrt{4-y^2}}$

$\int_0^2 \frac{y^4-8y^2+16}{2} \rightarrow \frac{y^5}{10} - \frac{4y^3}{3} - 8y \Big|_0^2 = \boxed{\frac{128}{15}}$

15.4

1.) $f(x,y) = \sqrt{x^2+y^2}$, $x^2+y^2 \le 2$

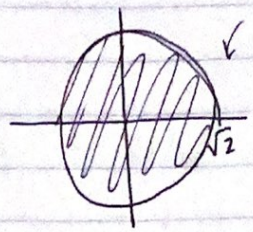
$r^2 = x^2+y^2$

$f(r,\theta) = \sqrt{r^2} = r$

$x^2+y^2 = r^2$

$r^2 \le 2$

$\sqrt{2} \le r \le \sqrt{2}$



↙ horrible circle skills, lorry :

$$\iint_D \sqrt{x^2+y^2}\, dA$$

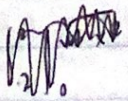
$r\cos\theta^2 + r\sin\theta^2 \le 2$

$r^2\cos^2\theta + \sin^2\theta \le 2$

$r^2(1) \le 2$

5.) $f(x,y) = y(x^2+y^2)^{-1}$

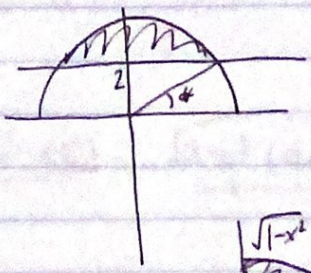
$r\sin\theta \ge \frac{1}{2}$



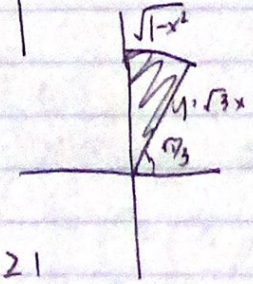
$r \le 1$

$r\sin\theta(r^2)^{-1}$

$\frac{\sin\theta}{r}$ $\sqrt{3} - \frac{\pi}{3}$?



9.) $\int_0^{1/2} \int_{\sqrt{3}x}^{\sqrt{1-x^2}} x \, dy\, dx$



$\sqrt{3}r\cos\theta \le y \le \sqrt{1-r^2\cos^2\theta}$

$= \frac{1}{3}\left(1 - \frac{\sqrt{3}}{2}\right)$

19.) $f(x,y) = x - y$ $\quad x^2+y^2 \le 1$ $\quad x+y \ge 1$

$r\cos\theta - r\sin\theta$

$r(\cos\theta - \sin\theta)$

$r = 1$

0

27) $f(x,y,z) = x^2+y^2$

bounds $r^2 \le 9$

bounds $-3 \le r \le 3$

$\tan(\theta) \frac{y}{x}$

$\int_0^5 r^2\,dz = r^2 z \Big|_0^5 \to 5r^2$

$\frac{5(r^4)\pi}{2} \Rightarrow \frac{5(3)^4\pi}{2} = \frac{5(81)\pi}{2} = \boxed{\frac{405\pi}{2}}$

31.) $f(x,y,z) = z$

$\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_{\sqrt{9-y^2}}^{9} z\,dz\,dx\,dy \to \frac{z^2}{2}\Big| \to \frac{-y^4+2x^2y^2+x^4-81}{2}$

$\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \frac{-y^4+2x^2y^2+x^4-81}{2} \to$

$\int_{-3}^{3} \frac{-\sqrt{9-y^2}(8y^4+36y^2-972)}{15} = \frac{-\sqrt{9-y^2}(4y^5+18y^3-1701y)-10935\arcsin(\frac{y}{3})}{4}$

$\boxed{243\pi}$

41.) $f(x,y,z) = x^2+y^2 \quad \rho \le 1$

$\rho^2 = x^2+y^2+z^2$

$\rho = \sqrt{r^2+z^2}$ $\quad r^2\,dr$

$\rho = r$

$r \le 1$

$r\cos\theta = \sqrt{1-y^2}$

$r^2\cos^2\theta = 1-y^2$

$r^2\cos^2\theta = 1-r^2\sin^2\theta$

$r^2(\cos^2\theta+\sin^2\theta) = 1$

$r^2 = 1$

$r = 1$

51.) $f(x,y,z) = z$

$z = \rho\cos\phi$

$\int_1^2\int_0^{\pi/2}\int_0^{\pi/3} \rho\cos\phi\,d\theta\,d\phi\,d\rho$

$\int_0^{\pi/3} \rho\cos\phi\,d\theta = \cos(\phi)\rho\theta\Big|_0^{\pi/3}$

$\int_0^{\pi/2} \frac{\pi}{3}\rho\cos(\phi)\,d\phi \to \frac{\pi\rho\sin(\phi)}{3}\Big|_0^{\pi/3}$

$\int_1^2 \frac{\pi\rho}{3} \to \frac{\pi\rho^2}{6} = \boxed{\frac{7}{2}}$