10/31/20 15.3 Triple Integrals

15.3 #3, 5, 7, 9, 11, 13, 15, 17

3) $f(x,y,z) = xe^{y-2z}$ $\quad 0 \le x \le 2,\ 0 \le y \le 1,\ 0 \le z \le 1.$

$\int_0^2\int_0^1\int_0^1 xe^{y-2z}\, dz\, dy\, dx$

$\int_0^2\int_0^1 -\frac{1}{2}(xe^{y-2z})\Big|_0^1\, dy\, dx$

$\int_0^2\int_0^1 \frac{xe^{y}}{2} - \frac{xe^{y-2}}{2}\, dy\, dx$

$\int_0^2 \left(\frac{xe^{1}}{2} - \frac{xe^{-1}}{2}\right) dx$

$\frac{1}{2}(2e - \frac{2}{e}) = e - \frac{1}{e}$

5) $f(x,y,z) = \int_0^3\int_0^3\int_0^1 (x-y)(y-z)\, dx\, dy\, dz$

$xy - xz - y^2 + yz$

$\int_0^3\int_0^3 \left(\frac{y}{2} - \frac{z}{2} - y^2 + yz\Big|_0^1\right) dy\, dz$

$\int_0^3 \left(\frac{9}{4} - \frac{3z}{2} - 9 + \frac{9z}{2}\Big|_0^3\right) dz$

$\frac{9}{4}z - \frac{3z^2}{4} - 9z + \frac{9z^2}{6}\Big|_0^3$

$= -\frac{27}{4}$

7) $f(x,y,z) = (x+z)^3 \quad [0;a] \times [0,b] \times [0,c]$

$$\int_0^a \int_0^b \int_0^c (x+z)^3 \, dx\, dy\, dz.$$

$$\int_0^a \int_0^b \frac{(c+z)^4}{4} - \frac{z^4}{4} \, dy\, dz.$$

$$\int_0^a \frac{b(c+z)^4}{4} - \frac{bz^4}{4} \Big|_0^b \, dz$$

$$\frac{b(c+z)^5}{20} - \frac{bz^5}{20} \Big|_0^a$$

$$\frac{b(c+a)^5}{20} - \frac{b(a^5)}{20}$$

9) $f(x,y,z) = x+y$

$$\int_0^1 \int_0^x \int_y^x x+y. \; dz\, dy\, dx$$

$$\int_0^1 \int_0^x \left(xz+yz \Big|_y^x\right) dy\, dx$$

$$\int_0^1 \int_0^x x\cancel{y}^2 + xy - (\cancel{x}y) - y^2 \, dy\, dx$$

$$x^2y + \frac{xy^3}{3} \Big|_0^x$$

$$\int_0^1 x^3 - \frac{x^3}{3} \, dx$$

$$\frac{x^4}{4}-\frac{x^4}{12}\Big|_0^1 = \frac{1}{4}-\frac{1}{12} = \frac{1}{6}$$

11) $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^1 xyz \; dz\,dy\,dx$

$$\int_0^1\int_0^{\sqrt{1-x^2}} \frac{xyz^2}{2}\Big|_0^1 \, dy\,dz = \frac{xy}{2}$$

$$\int_0^1 \frac{xy^2}{4}\Big|_0^{\sqrt{1-x^2}} dx \qquad x(1-x^2)$$

$$\int_0^1 \frac{x-x^3}{4}\,dx = \frac{x^2}{8}-\frac{x^4}{16}\Big|_0^1 =$$

$$\frac{1}{8}-\frac{1}{16} = \frac{1}{16}$$

13) $f(x,y,z) = e^z$, $W = x+y+z \le 1$, $x \ge 0, y \ge 0, z \ge 0$.

$$\int_0^{1-x-y} e^z\,dz$$

$$\int_0^{1-y} e^{1-x-y} - 1 \; dy$$

15) $z = \sqrt{9-x^2-y^2}$

① $\int_0^{\sqrt{9-x^2-y^2}} z\,dz$

$\frac{z^2}{2}$ $\frac{9-x^2-y^2}{2}$

$\int_0^x \frac{9-x^2-y^2}{2}\,dy$ $\frac{9y}{2} - \frac{x^2y}{2} - \frac{y^3}{6}\Big|_0^x$

$\frac{9x}{2} - \frac{x^3}{2} - \frac{x^3}{6}$

$\frac{9x^2}{18} - \frac{x^4}{8} - \frac{x^4}{24}\Big|_0^1 = \frac{9}{18} - \frac{1}{8} - \frac{1}{24} = \frac{1}{3}$

17) $f(x,y,z) = x$, $z = 8-2x^2-y^2$.

$\int_0^1\int_0^y\int_0^{8-2x^2-2y^2} x\,dz\,d\cancel{x}y\,dx$

$xz\Big|_0^{8-2x^2-2y^2}$

$\int_0^1\int_0^y x(8-2x^2-2y^2)\,dy\,dx$

$\int_0^1 8xy - 2x^3y - \frac{2}{3}y^3\,dx.$

④

$4x^2y - \frac{2}{3}x^3y - \frac{2}{3}y^3x$

$4y - \frac{2}{3}y - \frac{2}{3}y^3$

10/31/20 15.4 Integration with Polar.

15.4) # 1, 5, 9, 19, 27, 31, 47, 51

1) $f(x,y) = \sqrt{x^2+y^2}$ $\quad x^2+y^2 \le 2 \quad 0 \le r \le \sqrt{2}$

$\int_0^{2\pi}\int_0^{\sqrt{2}} r^2 \, dr \, d\theta \quad x^2+y^2 = r^2 \quad 0 \le \theta \le 2\pi$

$\frac{r^3}{3}\Big|_0^{\sqrt{2}} \quad \int_0^{2\pi} \frac{2\sqrt{2}}{3} \, d\theta$

$2\sqrt{2}\theta/3 \Big|_0^{2\pi} \rightarrow \frac{4\pi\sqrt{2}}{3}$

5) $f(x,y) = y(x^2+y^2)^{-1}, \quad y \ge \frac{1}{2}, \quad x^2+y^2 \le 1$

$0 \le r \le 1$

$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\int_0^1 \sin\theta \, dr \, d\theta$

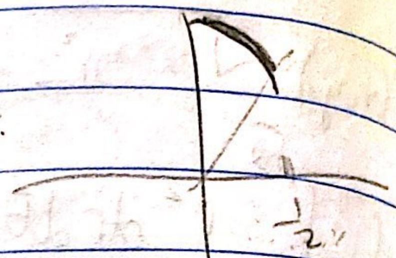
$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin\theta \, d\theta$

$-\cos\theta \Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = \sqrt{3}$

$\int$ Top circle r=1. → ~~circle~~ right ~~half~~ circle, r = 3

9) $\int_0^{\frac{1}{2}}\int_{x\sqrt{3}}^{\sqrt{1-x^2}} \sqrt{x^2+y^2}\,^2\, x\,dy\,dx.$

$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\int_0^1 r^2\cos\theta\,dr\,d\theta.$

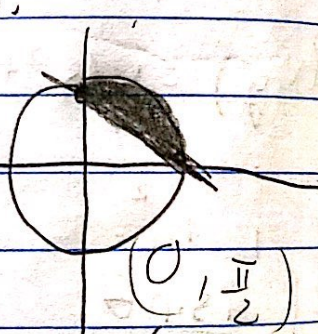


$\frac{1}{3}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\cos\theta}{3}\,d\theta = 0.045.$

19) $f(x,y) = x-y \quad x^2+y^2 \le 1, \quad (x-1)^2+y^2 \le 1.$ r=1

$\int_0^{\frac{\pi}{2}}\int_0^1 (r\cos\theta - r\sin\theta)\,r\,dr\,d\theta. \quad x+y \ge 1.$

$\int_0^{\frac{\pi}{2}} \frac{1^3}{3}\cos\theta - \frac{1}{3}\sin\theta\,d\theta.$



$= 0$

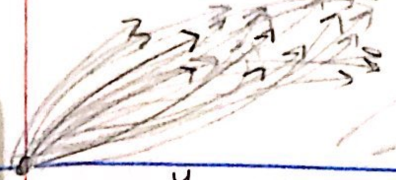
27) $f(x,y,z) = x^2 + y^2 \quad (r^2)$

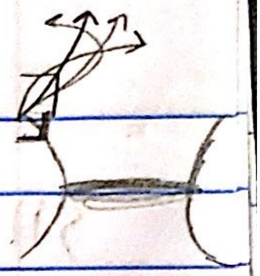
$x^2+y^2 \le 9 \quad (0,3)$ Polar.

$0 \le z \le 5 \rightarrow 0 \le \theta \le 2\pi$

$0 \le z \le 5$

$\int_0^5\int_0^{2\pi}\int_0^3 r^3\,dr\,d\theta\,dz.$





$\frac{r^4}{4} \rightarrow \int_0^5 \int_0^{2\pi} \frac{81}{4}\, d\theta\, dz$

$\frac{81}{4} \cdot (10\pi) = \frac{405\pi}{2}$

31) $f(x,y,z) = z$. $x^2+y^2 \le z \le 9$.

$\int_0^3 \int_0^{2\pi} \int_{r^2}^{9} z\, r\, dz\, d\theta\, dr$ $\quad r \le 3 \le 3$

$r^2 \le z \le 9$

$r \cdot \frac{z^2}{2}\Big|_{r^2}^{9}$

$\iint_0^{2\pi} \frac{81r}{2} - \frac{r^5}{2}$

$\frac{81r(2\pi)}{2} - r^5(\pi$

$\int_0^3 111 \quad dr = 243\pi$

47) $f(x,y,z) = x^2 + y^2$; $\rho \le 1$

$0 \le \theta \le 2\pi$

$0 \le \phi \le \pi$

$\int_0^{2\pi} \int_0^{\pi} \int_0^{1} (\rho\cos\theta \sin\phi)^2 + (\rho \sin\theta \sin\phi)^2$

51) $0 \le \theta \le \frac{\pi}{3}$ $0 \le \phi \le \frac{\pi}{2}$, $1 \le \rho \le 2$

$f(x,y,z) = z$

$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_1^2 (e\cos\phi) e^2 \sin\phi \, de \, d\phi \, d\theta$

$e^3 \cos\phi \sin\phi \, de \, d\phi \, d\theta$

$\int_1^2 e^3 \, de \int_0^{\frac{\pi}{2}} \frac{1}{2} \sin(2\phi) \, d\phi \int_0^{\frac{\pi}{3}} d\theta$

$= \frac{\pi}{3} \cdot \left(-\frac{1}{4} \cos 2\phi \Big|_0^{\frac{\pi}{2}} \right) \left(\frac{e^4}{4} \right) = \frac{5\pi}{6}$