15.3 [3, 5, 7, 9, 11, 13, 15, 17]
15.4 [1, 5, 9, 19, 27, 31, 47, 51]

15.3

3) $f(x,y,z) = xe^{y-2z}$; $0 \le x \le 2$, $0 \le y \le 1$, $0 \le z \le 1$

$\int_0^2 \int_0^1 \int_0^1 xe^{y-2z}\,dz\,dy\,dx \rightarrow ① \int_0^1 xe^{y-2z}\,dz = xe^y \int_0^1 e^{-2z}\,dz$ $\left\{ \begin{array}{l} u = -2z \rightarrow [0,-2] \\ du = -2dz \end{array} \right.$

$\frac{xe^y}{-2}\left[e^u \Big|_0^{-2} \right] = \frac{xe^y}{-2}\left(\frac{1}{e^2} - 1\right) \rightarrow ② \int_0^1 \frac{xe^y}{-2}\left(\frac{1}{e^2} - 1\right) dy = \frac{-x}{2}\left(\frac{1}{e^2} - 1\right) \int_0^1 e^y\,dy =$

$-\frac{x}{2}\left(\frac{1}{e^2} - 1\right)(e-1) \rightarrow ③ \int_0^2 -\frac{x}{2}\left(\frac{1}{e^2} - 1\right)(e-1)\,dx = \frac{-1}{2}\left(\frac{1}{e^2} - 1\right)(e-1)\int_0^2 x\,dx$

$\boxed{-\frac{1}{2}\left(\frac{1}{e^2} - 1\right)(e-1)(2)} = (1 - e^{-2})(e-1)$

5) $f(x,y,z) = (x-y)(y-z) = xy - xz - y^2 + yz$; $[0,1] \times [0,3] \times [0,3]$

$\int_0^1 \int_0^3 \int_0^3 xy - xz - y^2 + yz \; dz\,dy\,dx \rightarrow ① \int_0^3 (xy - xz - y^2 + yz)\,dz = xyz - \frac{xz^2}{2} - y^2 z + \frac{yz^2}{2} \Big|_0^3$

$3xy - \frac{9x}{2} - 3y^2 + \frac{9y}{2} \rightarrow ② \int_0^3 \left(3xy - \frac{9x}{2} - 3y^2 + \frac{9y}{2}\right) dy = \frac{3xy^2}{2} - \frac{9xy}{2} - y^3 + \frac{9y^2}{4} \Big|_0^3 =$

$\frac{27x}{2} - \frac{27x}{2} - 27 + \frac{81}{4} \rightarrow \int_0^1 \left(\cancel{\frac{27x}{2}} - \cancel{\frac{27x}{2}} - 27 + \frac{81}{4}\right) dx = -27x + \frac{81}{4}x \Big|_0^1$

$-27 + \frac{81}{4} = -6.75$

7) $f(x,y,z) = (x+z)^3$; $[0,a] \times [0,b] \times [0,c]$

$\int_0^b \int_0^c \int_0^a (x+z)^3\,dx\,dz\,dy \rightarrow ① \int_0^a (x+z)^3\,dx = \left\{ \begin{array}{l} \text{let } u = x+z \\ du = dx \end{array} \right\} \rightarrow \int_{x=0}^{x=a} u^3\,du \rightarrow$

$\frac{u^4}{4}\Big|_{x=0}^{x=a} = \frac{(x+z)^4}{4}\Big|_0^a = \frac{(a+z)^4}{4} - \frac{z^4}{4} \rightarrow \int_0^c \left(\frac{(a+z)^4}{4} - \frac{z^4}{4}\right) dz =$

$\frac{(a+z)^5}{20} - \frac{z^5}{20}\Big|_0^c = \frac{(a+c)^5}{20} - \frac{c^5}{20} \rightarrow ③ \int_0^b \frac{(a+c)^5}{20} - \frac{c^5}{20}\,dy = \frac{b}{20}\left[(a+c)^5 - c^5\right]$

9) $f(x,y,z) = x + y$; $W: y \le z \le x$, $0 \le y \le x$, $0 \le x \le 1$

$$\int_0^1\int_0^x\int_y^x x+y \, dz\,dy\,dx \Rightarrow (1)\int_y^x (x+y)\,dz = (x+y)(x-y) = (2)\int_0^x x^2-y^2\,dy = x^2y-\frac{y^3}{3}\Big|_0^x$$

$$x^3-\frac{x^3}{3} \Rightarrow (3)\int_0^1 \frac{x^3-x^3}{3} = \frac{x^4}{4}-\frac{x^4}{12}\Big|_0^1 = \frac{1}{4}-\frac{1}{12}=\frac{1}{6}$$

11) $f(x,y,z)=xyz$; $W: 0\le z\le 1,\ 0\le y\le\sqrt{1-x^2},\ 0\le x\le 1$

$$\int_0^1\int_0^1\int_0^{\sqrt{1-x^2}} xyz\,dy\,dx\,dz \Rightarrow (1)\int_0^{\sqrt{1-x^2}} xyz\,dy = xz\int_0^{\sqrt{1-x^2}} y\,dy = xz\left(\frac{y^2}{2}\Big|_0^{\sqrt{1-x^2}}\right)$$

$$xz\left(\frac{1-x^2}{2}\right) \Rightarrow \int_0^1 xz\left(\frac{1-x^2}{2}\right)dx = z\int_0^1 x\left(\frac{1-x^2}{2}\right) = z\int_0^1 \frac{x-x^3}{2}\,dx$$

$$\frac{z}{2}\left(\frac{x^2}{2}-\frac{x^4}{4}\Big|_0^1\right) = \frac{z}{2}\left(\frac{1}{2}-\frac{1}{4}\right) = \int_0^1 \frac{z}{8}\,dz = \frac{1}{8}\,\frac{z^2}{2}\Big|_0^1 = \frac{1}{2}\left(\frac{1}{8}\right)=\frac{1}{16}$$

13) $f(x,y,z)=e^z$; $W: x+y+z\le 1,\ x\ge 0,\ y\ge 0,\ z\ge 0$

$x+y+z\le 1$ $\quad 0\le z\le 1-x-y$
$x=0$ $\quad 0\le y\le 1-x$
$y=0$ $\quad 0\le x\le 1$
$z=0$

$$\int_0^1\int_0^{1-x}\int_0^{1-x-y} e^z\,dz \Rightarrow (1)\int_0^{1-x-y} e^z\,dz = e^{1-x-y}-1 \Rightarrow \int_0^{1-x}(e^{1-x-y}-1)\,dy \quad \begin{cases} u=1-x-y \\ du=-dy \end{cases}$$

$$u=[1-x,\ 1-x-(1-x) = \cancel{1}-\cancel{x}-\cancel{1}+\cancel{x}=0] \to -\int_{1-x}^0 e^u\,du - \int_0^{1-x} dy = -\left(e^u\Big|_{1-x}^0\right)-(1-x)$$

$$-(1-e^{1-x})-(1-x) = e^{1-x}-1-1+x = x+e^{1-x}-2 \Rightarrow \int_0^1 (x+e^{1-x}-2)\,dx$$

$$\frac{x^2}{2}\Big|_0^1 - \left(2x\Big|_0^1\right) + (e-1) = \frac{1}{2}-2+e-1 = \boxed{e-\frac{5}{2}}$$

$$\int_0^1 e^{1-x}\,dx \quad \text{let } u=1-x \Rightarrow [1,0] \quad -\int_1^0 e^u\,du = -(1-e)$$
$$du=-dx \Rightarrow -du=dx$$

15) $f(x,y,z)=z$ $\quad W: x^2+y^2+z^2=9,\ x=1,\ y=0,\ x=y$

$W: 0\le z\le\sqrt{9-x^2-y^2},\ 0\le y\le x,\ 0\le x\le 1$

$$\int_0^1\int_0^x\int_0^{\sqrt{9-x^2-y^2}} z\,dz\,dy\,dx \Rightarrow (1)\int_0^{\sqrt{9-x^2-y^2}} z\,dz = \frac{z^2}{2}\Big|_0^{\sqrt{9-x^2-y^2}} = \frac{9-x^2-y^2}{2}$$

$$\int_0^x \frac{9-x^2-y^2}{2}\,dy = \frac{1}{2}\left[9y - x^2y - \frac{y^2}{2}\Big|_0^x\right] = \frac{1}{2}\left[9x - x^3 - \frac{x^2}{2}\right]$$

$$\frac{1}{2}\int_0^1 9x - x^3 - \frac{x^2}{2}\,dx = \frac{1}{2}\left[\frac{9x^2}{2} - \frac{x^4}{4} - \frac{x^3}{6}\Big|_0^1\right] = \frac{1}{2}\left[\frac{9}{2} - \frac{1}{4} - \frac{1}{6}\right] = 2\frac{1}{12}$$

17) $f(x,y,z) = x$ W: $x \geq 0$, $y \geq 0$, $z \geq 0$ & $z = y^2$ $z = 8 - 2x^2 - y^2$

$y^2 \leq z \leq 8 - x^2 - y^2$, $0 \leq y \leq \sqrt{8-2x^2}$, $0 \leq x \leq 2$

$y^2 = 8 - 2x^2 - y^2$
$2y^2 = 8 - 2x^2$
$y^2 = 4 - x^2$
$y = \sqrt{4 - x^2}$
$0 = \sqrt{4 - x^2}$
$0 = 4 - x^2$
$-4 = -x^2$
$4 = x^2$

$$\int_0^2 \int_0^{\sqrt{8-2x^2}} \int_{y^2}^{8-x^2-y^2} x\,dz\,dy\,dx = \frac{128}{15}$$

*Use Maple

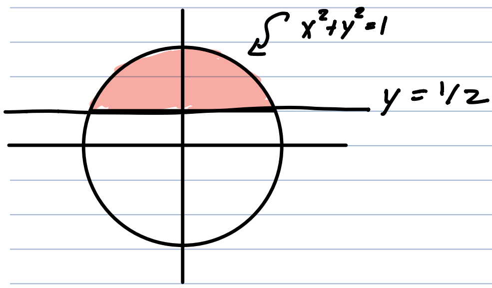
15.4 [1,5,9,19,27,31,47,51]

1) $f(x,y)=\sqrt{x^2+y^2}$, $x^2+y^2\le 2$ W: $0\le\theta\le 2\pi$, $0\le r\le\sqrt{2}$

$x^2+y^2=2 \rightarrow r^2=2$

$\int_0^{2\pi}\int_0^{\sqrt{2}} r\cdot r\,dr\,d\theta \Rightarrow ①\int_0^{\sqrt{2}} r^2 dr \rightarrow \frac{r^3}{3}\Big|_0^{\sqrt{2}} = \int_0^{2\pi}\frac{2\sqrt{2}}{3}d\theta = \frac{4\sqrt{2}\pi}{3}$

5) $f(x,y)=y(x^2+y^2)^{-1}$; $y\ge 1/2$ $x^2+y^2\le 1$



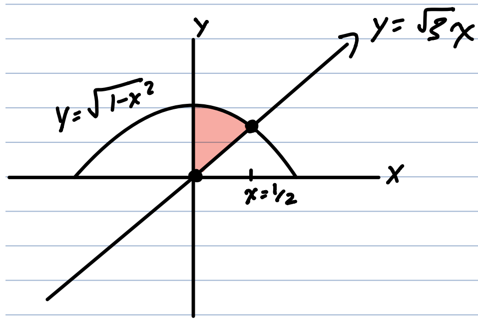
$D_{cartes.}=\{(x,y): 1/2\le y\le 1, -\frac{\sqrt{3}}{2}\le x\le\sqrt{1-y^2}$

$D_{polar.}=\{(r,\theta): \frac{\pi}{6}\le\theta\le\frac{5\pi}{6}\ \ \frac{1}{2\sin\theta}\le r\le 1$

$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\int_{1/2\sin\theta}^{1} \sin\theta\,dr\,d\theta \approx .685$ * Using Maple

9) $\int_0^{1/2}\int_{\sqrt{3}x}^{\sqrt{1-x^2}} x\,dy\,dx$ $D_{cart}=\{(x,y)\mid 0\le x\le 1/2 \quad \sqrt{3}x\le y\le\sqrt{1-x^2}$

$D_{polar}=\{(r,\theta): \frac{\pi}{3}\le\theta\le\frac{\pi}{2}, \ 0\le r\le 1$



$x=0, x=1/2$ } $y=\sqrt{3}x$, $y=\sqrt{1-x^2}$

$y^2+x^2=1$ $y=\frac{\sqrt{3}}{2}$ $r\cos\theta=0$

$r=1$

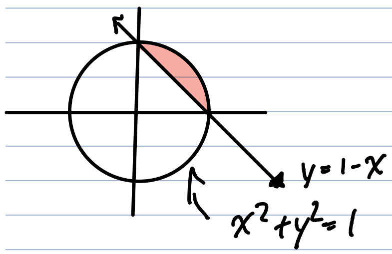
$r\sin\theta=\frac{\sqrt{3}}{2}\Rightarrow \sin\theta=\frac{\sqrt{3}}{2}$

$\theta=\frac{\pi}{3}$ or $\theta=\frac{\pi}{2}$

$\int_{\frac{\pi}{3}}^{\pi/2}\int_0^1 r^2\cos\theta\,dr\,d\theta \Rightarrow ①\int_0^1 r^2\cos\theta\,dr = \frac{r^3}{3}\cos\theta\Big|_0^1 = \frac{\cos\theta}{3}\Rightarrow\int_{\pi/3}^{\pi/2}\frac{\cos\theta\,d\theta}{3}$

$\frac{1}{3}\int_{\pi/3}^{\pi/2}\cos\theta\,d\theta = \frac{1}{3}\left(\sin\theta\Big|_{\pi/3}^{\pi/2}\right) = \left(1-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{3}\right)$

19) $f(x,y) = x - y;\ x^2+y^2 \leq 1,\ x+y \geq 1$



$D_{cart}: \{(x,y): 0 \leq x \leq 1,\ 1-x \leq y \leq \sqrt{1-x^2}\}$

$1-x = \sqrt{1-x^2}$
$(1-x)^2 = 1-x^2$
$1-2x+x^2 = 1-x^2$
$1-2x+2x^2 = 1$
$2x^2-2x = 0$
$x = 0$ or $x = 1$

$y = 0 \qquad y = 1$

$r\sin\theta \qquad r\sin\theta$

$D_{polar}: \{(r,\theta): 0 \leq r \leq 1 \quad 0 \leq \theta \leq \pi/2\}$

$$\int_0^{\frac{\pi}{2}} \int_0^1 r^2(\cos\theta - \sin\theta)\,dr\,d\theta \Rightarrow ①\int_0^1 r^2(\cos\theta-\sin\theta)\,dr = \frac{r^3}{3}(\cos\theta-\sin\theta)\Big|_0^1$$

$$\frac{1}{3}\int_0^{\pi/2} \cos\theta - \sin\theta\,d\theta = \frac{1}{3}\left(\sin\theta + \cos\theta\Big|_0^{\pi/2}\right) = \frac{1}{3}\left[\left(\sin\left(\frac{\pi}{2}\right)+\cos\left(\frac{\pi}{2}\right)\right)-\left(\sin(0)+\cos(0)\right)\right]$$

$= 0$

27) $f(x,y,z) = x^2+y^2;\ x^2+y^2 \leq 9 \quad 0 \leq z \leq 5$

$D_{cyc}: \{(r,\theta,z) \mid 0 \leq \theta \leq 2\pi,\ 0 \leq r \leq 3,\ 0 \leq z \leq 5$

$$\int_0^{2\pi}\int_0^3\int_0^5 (r^2\cos^2\theta + r^2\sin^2\theta)r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^3\int_0^5 (\cos^2\theta+\sin^2\theta)r^3\,dz\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^3\int_0^5 r^3\,dz\,dr\,d\theta \Rightarrow ①\int_0^5 r^3\,dz = r^3 \cdot z\Big|_0^5 = \int_0^3 5r^3\,dr = \frac{5r^4}{4}\Big|_0^3 =$$

$$\int_0^{2\pi} \frac{405}{4}\,d\theta = \frac{810}{4}\pi$$

31) $f(x,y,z) = z;\ x^2+y^2 \leq z \leq 9 \quad D = \{(r,\theta,z) \mid 0 \leq \theta \leq 2\pi,\ 0 \leq r \leq 3,\ r^2 \leq z \leq 9$

$z = x^2+y^2$
$z = 9$

$$\int_0^{2\pi}\int_0^3\int_{r^2}^9 z\,dz\,dr\,d\theta = 243\pi$$

$9 = x^2+y^2$
$r = 3$

*Use Maple

47) $f(x,y,z)=x^2+y^2$, $\rho \le 1$

$x=\rho\sin\phi\cos\theta$ $y=\rho\sin\phi\sin\theta$ $z=\rho\cos\theta$ $r=\rho\sin\phi$

$D=\{(\theta,\phi,\rho) \mid 0\le\rho\le 1, 0\le\phi\le\pi, 0\le\theta\le 2\pi$

$\int_0^{2\pi}\int_0^{\pi}\int_0^{1} \rho^4 \sin^3\phi \, d\rho \, d\phi \, d\theta = \frac{8\pi}{15}$ * Use MAPLE

51) $\int_0^{\pi/3}\int_0^{\frac{\pi}{2}}\int_1^{2} \rho^3\cos\phi\sin\phi \, d\rho \, d\phi \, d\theta = \frac{5\pi}{8}$

* Use Maple