15.3

3. $f(x,y,z) = xe^{y-2z}$; $0 \le x \le 2$, $0 \le y \le 1$, $0 \le z \le 1$

$$= \int_0^2 \int_0^1 \int_0^1 xe^{y-2z}\, dz\, dy\, dx$$

$$= \frac{(e-1)^2(1+e)}{e^2}$$

5. $f(x,y,z) = (x-y)(y-z)$ $[0,1]\times[0,3]\times[0,3]$

$$= \int_0^1 \int_0^3 \int_0^3 (x-y)(y-z)\, dz\, dy\, dx$$

$$= -\frac{27}{4}$$

7. $f(x,y,z) = (x+z)^3$ $[0,a]\times[0,b]\times[0,c]$

$$= \int_0^a \int_0^b \int_0^c (x+z)^3\, dz\, dy\, dx = \int_0^c \int_0^b \int_0^a (x+z)^3\, dx\, dy\, dz$$

$$= \int_0^c \int_0^b \left(\frac{(z+a)^4}{4} - \frac{z^4}{4}\right) dy\, dz$$

$$= \int_0^c b\,\frac{(a+z)^4 - z^4}{4}\, dz$$

$$= \frac{b}{4}(a^4c + 2a^3c^2 + 2a^2c^3 + ac^4)$$

9. $f(x,y,z) = x+y$ W: $y \le z \le x$, $0 \le y \le x$, $0 \le x \le 1$

$$= \int_y^x \int_0^x \int_0^1 x+y\, dx\, dy\, dz$$

$$= \int_y^x \int_0^x \left(y+\frac{1}{2}\right) dy\, dz$$

$$= \int_y^x \left(\frac{1}{2}x + \frac{x^2}{2}\right) dz$$

$$= \frac{x^2+x}{2}x - \frac{x^2+x}{2}y$$

扫描全能王 创建

11. $f(x,y,z)=xyz$ $W: 0\le z\le 1,\ 0\le y\le\sqrt{1-x^2},\ 0\le x\le 1$

$$=\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^1 xyz\,dx\,dy\,dz$$

$$=\int_0^1\int_0^{\sqrt{1-x^2}} yz\,\frac{1}{2}\,dy\,dz$$

$$=\int_0^1 \left(\frac{z(-x^2+1)}{4}\right)dz$$

$$=\frac{1-x^2}{8}$$

13. $f(x,y,z)=e^z$ $W: x+y+z\le 1,\ x\ge 0,\ y\ge 0,\ z\ge 0$

$$=\int_0^1\int_0^{1-x}\int_0^{1-x-y} e^z\,dz\,dy\,dx$$

$0\le x\le 1$
$0\le y\le 1-x$
$0\le z\le 1-x-y$

$$=e-\frac{5}{2}$$

15. $0\le z\le\sqrt{9-x^2-y^2},\ 0\le y\le x,\ 0\le x\le 1$

$$\int_0^1\int_0^x\int_0^{\sqrt{9-x^2-y^2}} z\,dz\,dy\,dx$$

$$=\int_0^1\int_0^x \left(\frac{9-x^2-y^2}{2}\right)dy\,dx$$

$$=\int_0^1 \frac{9x}{2}-\frac{x^3}{2}-\frac{x^3}{6}\,dx$$

$$=\frac{25}{12}$$

17. $0\le x\le 2,\ 0\le y\le\sqrt{4-x^2},\ y^2\le z\le 8-2x^2-y^2$

$$\int_0^2\int_0^{\sqrt{4-x^2}}\int_{y^2}^{8-2x^2-y^2} x\,dz\,dy\,dx$$

$$=\int_0^2\int_0^{\sqrt{4-x^2}} x(8-2x^2-y^2-y^2)\,dy\,dx$$

$$=\frac{4}{3}\int_0^2 x(4-x^2)^{\frac{3}{2}}\,dx$$

$$=\frac{128}{15}$$

扫描全能王 创建

15.4

1. $f(x,y)=\sqrt{x^2+y^2}$, $x^2+y^2\le 2$

$$\int_0^{2\pi}\int_0^{\sqrt{2}} r^2 r\,dr\,d\theta$$

$$=\frac{4\sqrt{2}\pi}{3}$$

5. $f(x,y)=y(x^2+y^2)^{-1}$; $y\ge\frac{1}{2}$, $x^2+y^2\le 1$

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\int_0^1 r\sin\theta(r^2)^{-1}r\,dr\,d\theta$$

$$=\sqrt{3}-\frac{\pi}{3}$$

9. $\int_0^{\frac{1}{2}}\int_{\sqrt{3}x}^{\sqrt{1-x^2}} x\,dy\,dx$ $\sqrt{3}x\le y\le\sqrt{1-x^2}$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\int_0^1 r\cos\theta r\,dr\,d\theta$$

$$=\frac{1}{3}-\frac{\sqrt{2}}{6}$$

19. $f(x,y)=x-y$; $x^2+y^2\le 1$, $x+y\ge 1$

$$\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}\int_0^1 r(\cos\theta-\sin\theta)r\,dr\,d\theta$$

$$=0$$

27. $f(x,y,z)=x^2+y^2$, $x^2+y^2\le 9$, $0\le z\le 5$

$$\int_0^5\int_0^{2\pi}\int_0^3 r^2 r\,dr\,d\theta\,dz$$

$$=\frac{405\pi}{2}$$

31. $f(x,y,z)=z$, $x^2+y^2\le z\le 9$

$$\int_0^3\int_0^{2\pi}\int_{r^2}^9 z\,r\,dz\,d\theta\,dr$$

$$=\frac{243\pi}{4}$$

扫描全能王 创建

47. $f(x,y,z) = x^2+y^2$; $\rho \le 1$

$$\int_0^{\pi}\int_0^{2\pi}\int_0^{5} \rho^2\sin^2\varphi\cos^2\theta + \rho^2\sin^2\varphi\sin^2\theta \, d\rho\, d\theta\, d\varphi$$

$$= \frac{8\pi}{15}$$

51. $f(x,y,z) = z$

$$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{3}}\int_{\sec\varphi}^{2} \rho\cos\varphi \, d\rho\, d\theta\, d\varphi$$

$$= \frac{5\pi}{8}$$

扫描全能王 创建