15.3

3. $f(x,y,z) = xe^{y-2z}$ $\quad 0 \le x \le 2,\ 0 \le y \le 1 \quad 0 \le z \le 1$

$\int_0^2\int_0^1\int_0^1 xe^{y-2z}\, dz\, dy\, dx$

$x\int_0^1 e^{y-2z}\, dz = x\left(\frac{-e^{y-2}+e^{y}}{2}\right)$

$\frac{x}{2}\int_0^1 -e^{y-2}+e^{y}\, dy = \frac{x}{2}(e^{-1}+e^{1}+e^{-2}-1)$ $\quad \int_0^2 \frac{x}{2}(e^{-1}+e^{1}+e^{-2}-1) =$

$\frac{x^2}{4}(e^{-1}+e^{1}+e^{-2}-1)\Big|_0^2 =$

$\frac{4}{4}(e^{-1}+e+e^{-2}-1)$

5. $\int_0^1\int_0^3\int_0^3 (x-y)(y-z)\, dz\, dy\, dx$

$\int_0^3 xy-y^2+yz-xz\, dz = xyz-zy^2+\frac{yz^2}{2}-\frac{xz^2}{2}\Big|_0^3 = 3xy-3y^2+\frac{9y}{2}-\frac{9x}{2}$

$\int_0^3 3xy-3y^2+\frac{9y}{2}-\frac{9x}{2}\, dy = \frac{3xy^2}{2}-y^3+\frac{9y^2}{4}-\frac{9xy}{2}\Big|_0^3 = 9x-9+6-\frac{27x}{2}$

$\int_0^1 9x-3-\frac{27x}{2}\, dx = \frac{9x^2}{2}-3x-\frac{27x^2}{4}\Big|_0^1 = \frac{9}{2}-\frac{3}{12}-\frac{27}{4} = -\frac{27}{4}$

7. $\int_0^a\int_0^b\int_0^c (x+z)^3\, dz\, dy\, dx$

$\int_0^c (x+z)^3\, dz = \frac{(x+z)^4}{4}\Big|_0^c = \frac{(x+c)^4}{4}-\frac{x^4}{4}$

$\frac{1}{4}\int_0^b (x+c)^4+x^4\, dy = y(x+c)^4+yx^4\Big|_0^b = \frac{b(x+c)^4+bx^4}{4}$

$\frac{1}{4}\int_0^a b(x+c)^4+bx^4\, dx = \frac{b}{4}\left(\frac{(x+c)^5}{5}+\frac{x^5}{5}\right)\Big|_0^a = \frac{b}{20}\left[(a+c)^5-a^5\right]-c^5$

9. $\int_0^1\int_0^x\int_y^x x+y\, dz\, dy\, dx$

$\int_y^x x+y\, dz = xz+yz\Big|_y^x = (x^2+yx)-(xy+y^2) = x^2-y^2$

$\int_0^x x^2-y^2\, dy = x^2y-\frac{y^3}{3}\Big|_0^x = x^3-\frac{x^3}{3}$

$\int_0^1 x^3-\frac{x^3}{3} = \frac{x^4}{4}-\frac{x^4}{12}\Big|_0^1 = \frac{1^3}{4}-\frac{1}{12} = \frac{1}{6}$

11. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^1 xyz \, dz\,dy\,dx =$

$\int_0^1 xyz \, dz = xy \frac{z^2}{2} \Big|_0^1 = \frac{xy}{2}$

$\frac{1}{2}\int_0^{\sqrt{1-x^2}} xy\,dy = \frac{xy^2}{4} \Big|_0^{\sqrt{1-x^2}} = \frac{x(1-x^2)}{4} = \frac{x-x^3}{4}$

$\frac{1}{4}\int_0^1 x - x^3 \, dx = \frac{x^2}{8} - \frac{x^4}{16} \Big|_0^1 = \frac{2}{8_{16}} - \frac{1}{16} = \frac{1}{16}$

13. e^z , $x+y+z \leq 1$, $x \geq 0$ $y \geq 0$ $z \geq 0$

$0 \leq z \leq 1-x-y$

$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} e^z \, dz\,dy\,dx$

$\int_0^{1-x-y} e^z \, dz = e^z \Big|_0^{1-x-y} = e^{1-x-y} - 1$

$\int_0^{1-x} e^{1-x-y} - 1 \, dy = -e^{1-x-y} - y \Big|_0^{1-x} = (-e^{(1-x)-(1-x)} - 1 - x) - (-e^{1-x})$

$= -2 - x + e^{1-x}$

$\int_0^1 -2 - x + e^{1-x} dx = -2x - \frac{x^2}{2} - e^{1-x} \Big|_0^1 = e - \frac{5}{2}$

15. $x=1, y=0, x=y$ $0 \leq x \leq 1$ $0 \leq y \leq x$ $1 \leq x \leq \sqrt{9-y^2-x^2}$

$\int_0^1 \int_0^x \int_1^{\sqrt{1-y^2-x^2}} z \, dz\,dy\,dx$

$\int_1^{\sqrt{9-y^2-x^2}} z \, dz = \frac{z^2}{2} \Big|_1^{\sqrt{9-y^2-x^2}} = \frac{9-y^2-x^2}{2} - \frac{1}{2}$

$\frac{1}{2}\int_0^x 8 - y^2 - x^2 \, dy = 8y - \frac{y^3}{3} - yx^2 = \frac{1}{2}(8x - \frac{x^3}{3} - x^3)$

$\frac{1}{2}\int_0^1 8x - \frac{x^3}{3} - x^3 \, dx = \frac{1}{2}\left(\frac{8x^2}{2} - \frac{x^4}{12} - \frac{x^4}{4}\right)\Big|_0^1 = \frac{1}{2}\left(\frac{8^{48}}{2_{12}} - \frac{1}{12} - \frac{1^{3}}{4_{12}}\right) = \frac{44}{24}$

17. $x \quad x \geq 0 \quad y \geq 0 \quad z \geq 0$ above $z = y^2$ below $z = 8 - 2x^2 - y^2$

$$y^2 = 8 - 2x^2 - y^2$$
$$2y^2 = 8 - 2x^2$$
$$y = \sqrt{4 - x^2}$$

$0 \leq z \leq 8 - 2x^2 - y^2 \qquad 0 \leq y \leq \sqrt{4 - x^2} \qquad 0 \leq x \leq 2$

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{8-2x^2-y^2} x \, dz \, dy \, dx$$

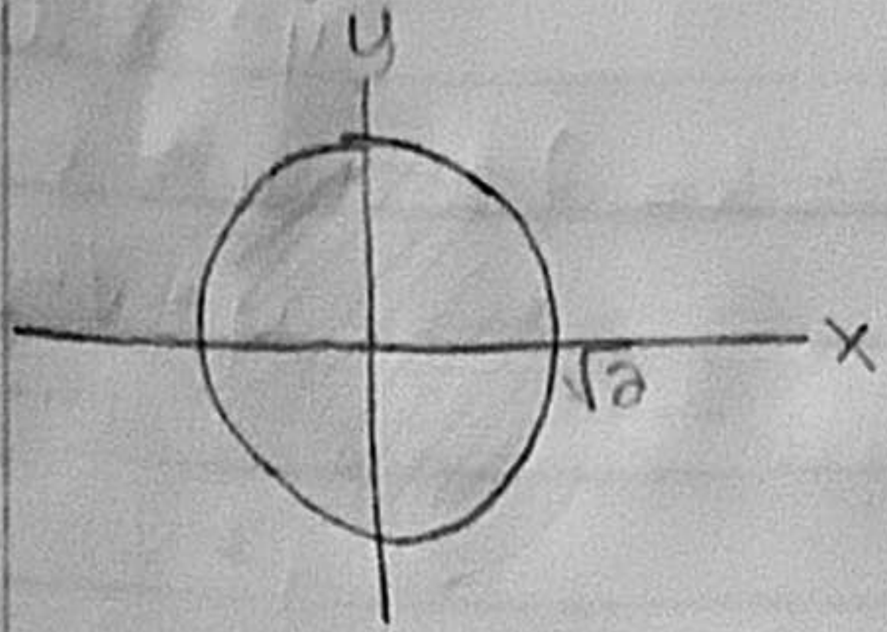
$$\int_0^{8-2x^2-y^2} x \, dz = xz \Big|_0^{8-2x^2-y^2} = x(8 - 2x^2 - y^2)$$

$$x \int_0^{\sqrt{4-x^2}} (8 - 2x^2 - y^2) \, dy = x\left(8y - 2yx^2 - \frac{y^3}{3}\right) \Big|_0^{\sqrt{4-x^2}} = x\left(8\sqrt{4-x^2} - 2x^2\sqrt{4-x^2} - \frac{(4-x^2)\sqrt{4-x^2}}{3}\right)$$

$$\int_0^2 x \cdot \sqrt{4-x^2} \cdot \left(8 - 2x^2 - \frac{4-x^2}{3}\right) dx = \frac{64}{15} \pm \frac{128}{15}$$

15.4

1. $f(x,y)=\sqrt{x^2+y^2}$



$x^2+y^2 \le 2$

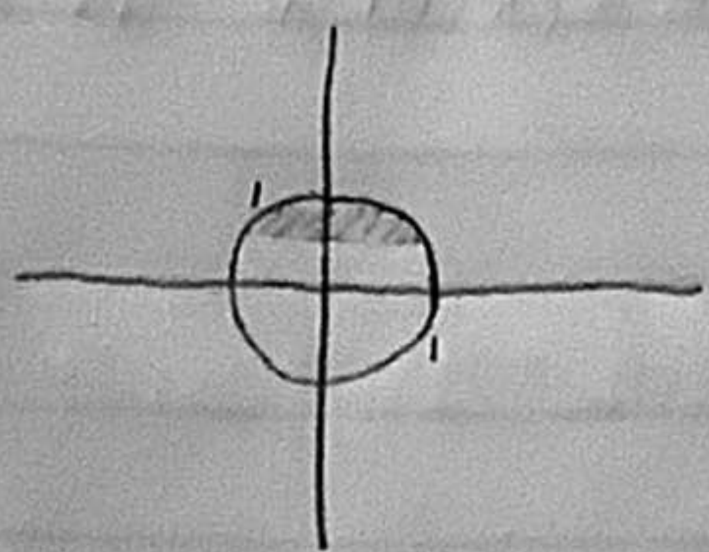
$r \le \sqrt{2}$

$\int_0^{2\pi}\int_0^{\sqrt{2}} r \cdot r\,dr\,d\theta = \frac{r^3}{3}\Big|_0^{\sqrt{2}} = \frac{2\sqrt{2}}{3}$

$\int_0^{2\pi} \frac{2\sqrt{2}}{3}\,d\theta = \frac{4\sqrt{2}\pi}{3}$

5. $f(x,y)=y(x^2+y^2)^{-1}$ $\quad y \ge \frac{1}{2}$ $\quad x^2+y^2 \le 1$

$r \le 1$



$r^{-1/2} \cdot r^2$

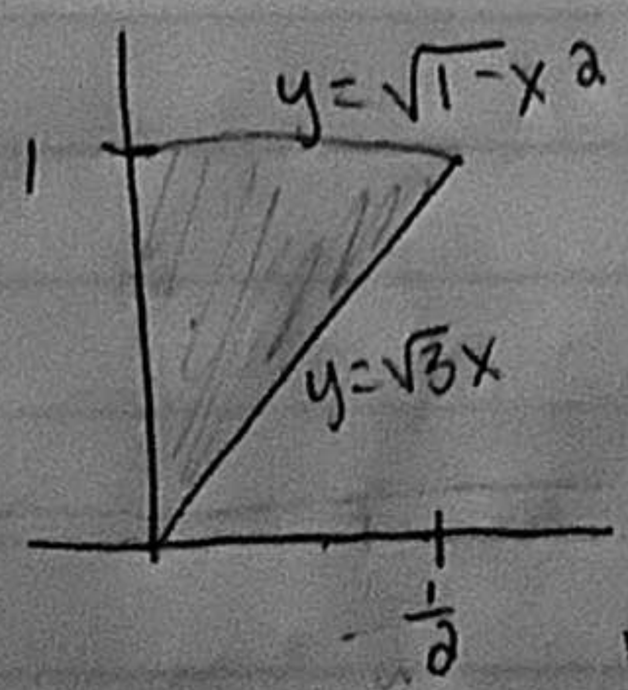
$\int_0^{\pi/2}\int_0^1 r\sin\theta(\sqrt{r})^{-1} \cdot r\,dr\,d\theta$

$\int_0^{\pi/2}\int_0^1 r^{3/2}\sin\theta\,dr\,d\theta = \frac{2r^{5/2}}{5}\Big|_0^1 \sin\theta$

$\frac{2}{5}\int_0^{\pi/2} \sin\theta\,d\theta = \frac{2}{5}\left(-\cos\theta\Big|_0^{\pi/2}\right) = \frac{2}{5}(-0-1)$

$= -\frac{2}{5}$

9. $\int_0^{1/2}\int_{\sqrt{3}x}^{\sqrt{1-x^2}} x\,dy\,dx$ $\quad \sqrt{1-x^2} \le y \le \sqrt{3}x$ $\quad 0 \le x \le \frac{1}{2}$



$r = 1^2 + \left(\frac{1}{2}\right)^2 = \frac{5}{4}$

$\int_0^{\pi/2}\int_0^{5/4} r\cos\theta \cdot r\,dr\,d\theta = \frac{r^3}{3}\Big|_0^{5/4} \cos\theta$

$\int_0^{\pi/2} \frac{125}{192}\cos\theta\,d\theta = \frac{125}{129}\sin\theta\Big|_0^{\pi/2}$

$= \frac{125}{129}$

19. $f(x,y)=x-y$ $\quad x^2+y^2 \le 1$ $\quad x+y \ge 1$

$r \le 1$

$\int_0^{\pi/2}\int_0^1 r\cos\theta - r\sin\theta\ r \cdot dr\,d\theta$

$\int_0^1 r^2(\cos\theta - \sin\theta)\,dr\,d\theta = \frac{r^3}{3}$

$\frac{1}{3}\int_0^{\pi/2} \cos\theta - \sin\theta\,d\theta = \frac{1}{3}\left(\cos\frac{\pi}{2} - \sin\frac{\pi}{2}\right) - (\cos 0 - \sin 0)$

$= \frac{1}{3}(-2) = -\frac{2}{3}$

27. $f(x,y,z) = x^2+y^2$ $\quad x^2+y^2 \le 9 \quad 0 \le z \le 5$

$0 \le r \le 3$

$$\int_0^{2\pi}\int_0^3\int_0^5 r^2 \, r\,dz\,dr\,d\theta = r^3 z \Big|_0^5 = 5r^3$$

$$5\int_0^3 r^3\,dr = \frac{r^4}{4} = \frac{405}{4}$$

$$\int_0^{2\pi} \frac{405}{4}\,d\theta = \frac{810\pi}{4} = \frac{405\pi}{2}$$

31. $f(x,y,z) = z$ $\quad x^2+y^2 \le z \le 9$

$r^2 \le z \le 9 \quad 0 \le r \le 3 \quad 0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_0^3\int_{r^2}^9 z\,r\,dz\,dr\,d\theta = \frac{z^2}{2}\cdot r\Big|_{r^2}^9 = \frac{81r}{2} - \frac{r^5}{2}$$

$$\int_0^3 \frac{81r}{2} - \frac{r^4}{2}\,dr = \frac{81r^2}{4} - \frac{r^5}{10}\Big|_0^3 = \left(\frac{81(3^2)}{4} - \frac{(3^5)}{10}\right) - 0 = \frac{729}{4} - \frac{243}{10}$$

$$= \frac{6318}{10}$$

$$\int_0^{2\pi} \frac{6318}{10}\,d\theta = 2\pi \cdot \frac{6318}{10} = 1263.6\pi$$

47. $f(x,y,z) = x^2+y^2 \quad 0 \le p \le 1, \quad 0 \le \theta \le 2\pi,$

$f(p,\theta,\phi) = (p\sin\phi\cos\theta)^2 + (p\sin\phi\sin\theta)^2$

$$\int_0\int_0^{2\pi}\int_0^1 p^2\sin^2\phi\,dp\,d\theta\,d\phi = \frac{p^3}{3}\Big|_0^1 = \frac{1}{3}\sin^2\phi$$

$$\int_0^{2\pi} \frac{1}{3}\sin^2\theta = \frac{2\pi}{3}\sin^2\theta \qquad \frac{4\pi}{3}\cos^2\theta\Big|_0^{\pi} = \frac{4\pi}{3}(-1-1) = \frac{8\pi}{3}$$

51. $$\int_0^{\pi/3}\int_0^{\pi/2}\int_1^2 p\cos\phi\,dp\,d\theta\,d\phi = \frac{p^2}{2}\Big|_1^2 = 2 - \frac{1}{2} = \frac{3}{2}\cos\phi$$

$$\int_0^{\pi/2} \frac{3}{2}\cos\phi\,d\theta = \frac{3\pi}{4}\cos\phi$$

$$\frac{3\pi}{4}\int_0^{\pi/3}\cos\phi\,d\phi = \sin\phi\Big|_0^{\pi/3} = \frac{\sqrt{3}}{2}\cdot\frac{3\pi}{4} = \frac{3\pi\sqrt{3}}{8}$$