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10/31/2020

15.3: 3, 5, 7, 9, 11, 13, 15, 17

3.

$$\int_0^2\int_0^1\int_0^1 xe^{y-2z}\,dz\,dy\,dx$$

$$\int_0^1 xe^{y-2z}\,dz$$

$$\frac{xe^{y-2z}}{-2}\Big|_0^1 \qquad e^{-1}+e+e^{-2}-1$$

$$\frac{xe^{y-2}}{-2}+\frac{xe^{y}}{2}=\frac{-xe^{y-2}+xe^{y}}{2}$$

$$\int_0^1 \frac{-xe^{y-2}+xe^{y}}{2}\,dy$$

$$\frac{-xe^{-1}+xe^{1}}{2}-\frac{-xe^{-2}+x}{2}=\frac{-xe^{-1}+xe+xe^{-2}-x}{2}$$

$$\int_0^2 \frac{-xe^{-1}+xe+xe^{-2}-x}{2} \to \frac{\frac{-x^2e^{-1}}{2}+\frac{x^2}{2}e+\frac{x^2e^{-2}}{2}-\frac{x^2}{2}}{2}-\frac{\frac{-2e^{-1}+2e+2e^{-2}-2}{2}}{2}=-e^{-1}+e+e^{-2}-1$$

5.

$$\int_0^1\int_0^3\int_0^3 xy - xz - y^2 + zy \, dz\,dy\,dx$$

$$xyz - \frac{xz^2}{2} - zy^2 + \frac{z^2y}{2} \Big|_0^3$$

$$\int_0^3 \left(3xy - \frac{9}{2}x - 3y^2 + \frac{9y}{2}\right) dy$$

$$\frac{3xy^2}{2} - \frac{9}{2}xy - y^3 + \frac{9y^2}{4} \Big|_0^3$$

$$\frac{27x}{2} - \frac{27x}{2} - 27 + \frac{81}{4} \Big|_0^1$$

$$-27 + \frac{81}{4} = -\frac{27}{4}$$

7. Answer, $\frac{b}{20}[(a+c)^5 - a^5 - c^5]$ $\frac{b(a+c)^5}{20} - \frac{b\,2c}{20}$

$$\int_0^a\int_0^b\int_0^c (x+z)^3 \, dz\,dy\,dx$$

$$\int_0^b \frac{(x+c)^4 - x^4}{4} dy \quad \frac{b}{4}\int_0^a (x+c)^4 - x^4 \, dx$$

$$b\left(\frac{(x+c)^4 - x^4}{4}\right) \qquad \frac{(x+c)^5}{5} - \frac{x^5}{5}$$

$$= \frac{b}{20}[(x+a)^5 - a^5 - c^5]$$

9.

$$\int_0^1 \int_0^x \int_y^x x+y \, dz\, dy\, dx$$

$$z(x+y) \Big|_y^x$$

$$x^2 + \cancel{yx} - \cancel{xy} - y^2$$

$$\int_0^x x^2 - y^2 \, dy$$

$$yx^2 - \frac{y^3}{3} \Big|_0^x$$

$$\int_0^1 x^3 - \frac{x^3}{3} \, dx$$

$$\frac{x^4}{4} - \frac{x^4}{12} \Big|_0^1$$

$$\frac{1}{4} - \frac{1}{12} = \frac{3}{12} - \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$$

$$\frac{1}{6}$$

11.

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^1 xyz \, dz \, dy \, dx$$

$$\frac{xzy^2}{2}\Big|_0^1$$

$$\int_0^{\sqrt{1-x}} \frac{xy}{2} \, dy$$

$$\frac{xy^2}{4}$$

$$\int_0^1 \frac{x(1-x^2)}{4} \, dx$$

$$\frac{1}{4}\int_0^1 x - x^3 \, dx$$

$$\frac{1}{4} \; \frac{x^2}{2} - \frac{x^4}{4}\Big|_0^1 \quad \frac{1}{16}$$

$$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

13. $\int_0^1 \int_0^1 \int_0^1 e^{z}\, dz\, dy\, dx$

$\int_0^1 e-1 \, dy$

$ye-y$

$e-1-1$

$\int_0^1 e-2y \, dx$

$xe-2x \Big|_0^1$

$e-2$

15

$$\int_0^3 \int_0^1 \int_0^{\sqrt{9-2y^2}} z \, dz \, dy \, dx$$

$$\frac{z^2}{2} \Big|_0^{\sqrt{9-2y^2}}$$

$$\int_0^1 \frac{9-2y^2}{2} \, dx$$

$$\frac{9}{2}y - \frac{y^3}{3} \Big|_0^1$$

$$\frac{9}{2} - \frac{1}{3}$$

$$\frac{18}{6} - \frac{2}{6} = \frac{16}{6} = \frac{8}{3}$$

$$\int_0^3 \frac{8}{3} \, dx$$

8

17. $\int_0^4 \int_0^{\sqrt{4-x^2}} \int_0^{y^2} x \, dz \, dy \, dx$

$y^2 = 8 - 2x^2 - y^2$

$\frac{2y^2}{2} = \frac{8 - 2x^2}{2}$

$y = \sqrt{4 - x^2}$

$xy \Big|_0^{y^2}$

$\int_0^{\sqrt{4-x^2}} xy^2 \, dy$

$x(4 - x^2)$

$\int_0^4 4x - x^3 \, dx$

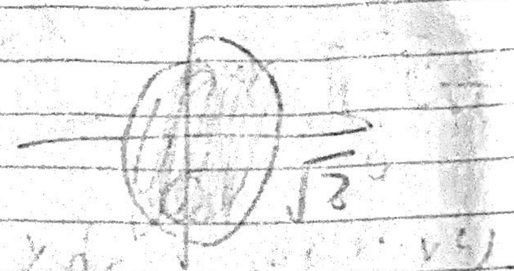
$2x^2 - \frac{x^4}{4} \Big|_0^4$ $\quad 32 - \frac{256}{4}$ $\quad 4\sqrt{256}$

$32 - 64$

$|-32| = 32$

15.4: 1, 5, 9, 19, 27, 31, 47, 51

1. $\int_0^{\pi}\int_{-\sqrt{2}}^{\sqrt{2}} r^2\, dr\, d\theta$

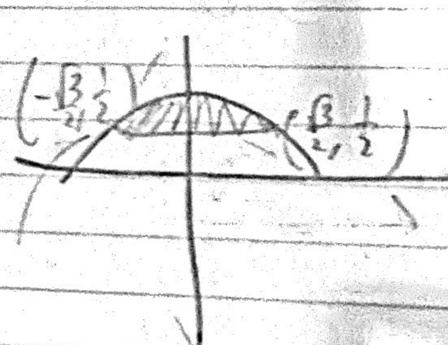


$\frac{r^3}{3}\Big|_{-\sqrt{2}}^{\sqrt{2}}$

$\int_0^{\pi} \frac{4\sqrt{2}}{3}\, d\theta$

$\frac{4\sqrt{2}}{3}\pi$

5. $\int_0^{\frac{\pi}{2}}\int_0^{2} r^{-1}\, r\sin\theta\, dr\, d\theta$



$\int_0^{2} \sin\theta\, d\theta$

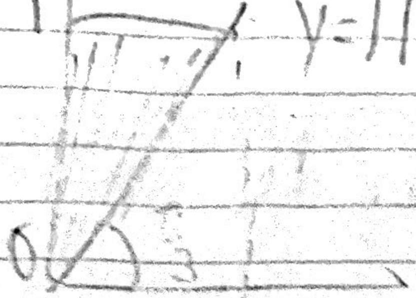
$\int_0^{\frac{\pi}{2}} 2\sin\theta\, d\theta$

$-2\cos\frac{\pi}{3} + 2\sin\frac{\pi}{3}$

$-2\cos\theta\Big|_{\frac{2\pi}{3}}^{\frac{\pi}{3}}$

$-\sqrt{3}+1$

9. $y=\sqrt{1-x^2}$

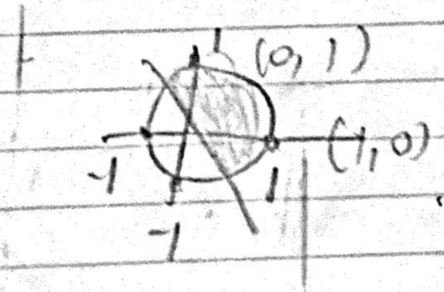


$$\int_0^{1}\int_{\sqrt{3x}}^{\sqrt{1-x^2}} x\,dy\,dx \qquad \frac{1}{3}\left(1-\frac{\sqrt{3}}{2}\right)$$

$$\int_{\frac{\pi}{3}}^{\pi}\int_0^1 r^2\cos\phi\, d\phi \qquad \frac{1}{3}\sin\phi\Big|_{\frac{\pi}{3}}^{\pi}$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{3}} \cos\phi\, d\theta \qquad 1-\frac{\sqrt{3}}{2}=\frac{1}{2}$$

19.



$y \geq 1-x \quad x^2 \leq 1-x^2$

$\sqrt{1-x^2} = (1-x)(1-x)$

$1-x^2 = 1-2x+x^2$

$$\int_{0}^{\frac{\pi}{2}}\int_0^1 r^2(\cos\theta-\sin\theta)\, dr\, d\theta \qquad -2x^2=-2x$$

$-2x^2+2x$

$x(-2x+1)$

$$\int_0^{\frac{\pi}{2}} \frac{1}{3}(\cos\theta-\sin\theta)$$

$$\frac{1}{3}(\sin\theta-\cos\theta)\Big|_0^{\frac{\pi}{2}} \qquad 0 \quad \frac{1}{3}$$

27. $\int_0^{2\pi}\int_0^3\int_0^5 r^3 \, dz \, dr \, d\theta$ $\quad \frac{81}{4} \cdot 5$ $\quad \frac{405}{4}$

$\int_0^3 5r^3 \, dr$

$\frac{5r^4}{4}\Big|_0^3$ $\quad \int_0^{2\pi} \frac{405}{4} \, d\theta$ $\quad \frac{405\pi}{2}$

31. $\int_0^{\pi}\int_0^3\int_0^{x^2+y^2} z \, dz \, dr \, d\theta$ $\quad \int_0^{\pi} \frac{243}{10} \, d\theta$

$\frac{z^2}{2}\Big|_0^{x^2+y^2}$ $\quad \frac{243\pi}{10}$

$\int_0^3 \frac{r^4}{2} \, dr$

$\frac{r^5}{10}\Big|_0^3$

x^2+y^2

47. $\int_0^{2\pi}\int_0^{2\pi}\int_{-1}^{1} \left(\rho^2\sin^2\theta\cos^2\theta + \rho^2\sin^2\varphi\sin^2\theta\right)\rho^2\sin\theta \, d\rho \, d\theta \, d\phi$

$\int_{-1}^{1} \rho^4\sin^3\phi\cos^2\theta + \rho^4\sin^2\phi\sin^3\phi \, d\phi$

$\frac{8\pi}{15}$

$\int_0^{2\pi} \frac{\rho^5}{5}\sin^3\phi\cos^2\theta + \frac{\rho^5}{5}\sin^3\phi\sin^2\theta \, d\phi$

$\int_0^{2\pi} \frac{2}{5}\sin^3\phi \, d\phi$ $\quad \frac{4\pi}{5}\int_0^{2\pi}\sin^3\phi \, d\phi$

$\int_0^{2\pi} \frac{4\pi}{5}\sin^3\phi \, d\phi$

$\frac{1}{3}\cos^3\phi - \cos\phi \Big|_0^{2\pi}$

51. $\frac{4\pi}{5}\left(\frac{2}{3}\right)$

$\frac{-\rho^4}{8}$ $\quad \frac{-16}{4}+\frac{1}{4}$ $\quad \frac{8\pi}{15}$ $\quad \frac{1}{3}+\frac{1}{3}=\frac{2}{3}$ $\quad -0$

$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{3}}\int_1^2 -\rho^3\frac{\sin(2\theta)}{2} \, d\rho \, d\theta \, d\phi$ $\quad -\sin(2\theta)\int_1^2 \rho^3$

$-\sin(2\theta)\frac{\rho^4}{4}$ $\quad \frac{\rho^4}{4}\Big|_1^2$

$\frac{16}{4}-\frac{1}{4}$ $\quad \frac{-15}{8}\int_0^{\pi}\sin(2\theta)\,d\theta$ $\quad \frac{1}{4}$

$\int_0^{\frac{\pi}{2}} \frac{15}{32} \, d\varphi$ $\quad \frac{\cos(2\theta)}{2}\Big|_0^{\frac{\pi}{3}}$

$-\frac{15}{8}\left(-\frac{1}{4}\right)$ $\quad \left(\frac{1}{2}\right)$

$\frac{15\pi}{64}$ $\quad 0$ $\quad \frac{15}{32}$ $\quad -\frac{1}{4}+\frac{2}{4}=\frac{1}{4}$ $\quad \frac{15}{8}$