15.3 Homework

(3) $f(x,y,z) = xe^{y-2z}$; $0 \le x \le 2$, $0 \le y \le 1$, $0 \le z \le 1$

$\rightarrow \int_0^2 \int_0^1 \int_0^1 xe^{y-2z}\, dz\, dy\, dx$

$\rightarrow \int_0^1 \left(xe^{y-2z}\right) dz = \frac{(e^2-1)xe^{y-2}}{2}$

$\rightarrow \int_0^1 \left(\frac{(e^2-1)xe^{y-2}}{2}\right) dy = \frac{e^{-2}(e-1)(e^2-1)x}{2}$

$\rightarrow \int_0^2 \left(\frac{e^{-2}(e-1)(e^2-1)x}{2}\right) dx = \boxed{(e-1)(1-e^{-2})}$

---

(5) $f(x,y,z) = (x-y)(y-2)$; $[0,1] \times [0,3] \times [0,3]$

$\rightarrow \int_0^1 \int_0^3 \int_0^3 (x-y)(y-2)\, dz\, dy\, dx$

$\rightarrow \int_0^3 (x-y)(y-2)\, dz = \frac{6y^2 + (-6x-9)y + 9x}{2}$

$\rightarrow \int_0^3 -\left(\frac{6y^2 + (-6x-9)y + 9x}{2}\right) dy = -\frac{27}{4}$

$\rightarrow \int_0^1 -\frac{27}{4}\, dx = \boxed{\frac{-27}{4}}$

---

(7) $f(x,y,z) = (x+z)^3$; $[0,a] \times [0,b] \times [0,c]$

$\rightarrow \int_0^a \int_0^b \int_0^c (x+z^3)\, dz\, dy\, dx$

$\rightarrow \int_0^c (x+z^3)\, dz = \frac{4cx+c^4}{4}$

$\rightarrow \int_0^b \left(\frac{4cx+c^4}{4}\right) dy = b\left(cx + \frac{c^4}{4}\right)$

$\rightarrow \int_0^a \left[b\left(cx + \frac{c^4}{4}\right)\right] dx = \boxed{\frac{abc(c^3+2a)}{4}}$

---

(11) $f(x,y,z) = xyz$; W: $z: [0,1]$, $y: [0, \sqrt{1-x^2}]$, $x: (0,1]$

$\rightarrow \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^1 xyz\, dz\, dy\, dx$

$\rightarrow \int_0^1 xyz\, dz = \frac{xy}{2}$

$\rightarrow \int_0^{\sqrt{1-x^2}} \frac{xy}{2}\, dy = \frac{x-x^3}{4}$

$\rightarrow \int_0^1 \left(\frac{x-x^3}{4}\right) dx = \boxed{\frac{1}{16}}$

---

(13) $f(x,y,z) = e^z$; W: $x+y+z \le 1$, $x \ge 0$, $y \ge 0$, $z \ge 0$

$\rightarrow \iint_W e^z\, dA = \boxed{e - \frac{5}{2}}$

---

(15) $f(x,y,z) = z$, $x = 1$, $y = 0$, $x = y$

→ $\iiint_W z \, dV = \boxed{\frac{25}{12}}$

---

(17) $f(x,y,z) = x$, $x \geq 0$, $y \geq 0$, $z \geq 0$

→ Above $z = y^2$ and below $z = 8 - 2x^2 - y^2$

→ $\iiint_W x \, dV = \boxed{\frac{128}{15}}$

15.4 Homework

① $f(x,y) = \sqrt{x^2+y^2}$, $x^2+y^2 \le 2$

→ $f(r,\theta) = r$

→ $0 \le r \le \sqrt{2}$

→ $0 \le \theta \le 2\pi$

→ $\int_0^{2\pi}\int_0^{\sqrt{2}} r^2\,dr\,d\theta =$ $\boxed{\frac{4\pi\sqrt{2}}{3}}$

---

⑤ $f(x,y) = y(x^2+y^2)^{-1}$, $y \ge \frac{1}{2}$, $x^2+y^2 \le 1$

→ $\frac{y}{x^2+y^2}$ ⇒ $\frac{r\sin\theta}{r^2}$ ⇒ $\frac{\sin\theta}{r}$

→ $\frac{\csc\theta}{2} \le r \le 1$, $\frac{\pi}{6} \le \theta \le \frac{5\pi}{6}$

→ $\int_{\pi/6}^{5\pi/6}\int_{\frac{\csc\theta}{2}}^{1} \frac{r\sin\theta}{r}\,dr\,d\theta =$ $\boxed{\sqrt{3}-\frac{\pi}{3}}$

---

⑨ $\int_0^{1/2}\int_{\sqrt{3}x}^{\sqrt{1-x^2}} x\,dy\,dx$; $0 \le y \le \frac{1}{2}$, $\sqrt{3}x \le x \le \sqrt{1-x^2}$

→ $y = \sqrt{3}x$ ⇒ $r\sin\theta = \sqrt{3}\,r\cos\theta$

→ $\tan\theta = \sqrt{3}$ ⇒ $\theta = \pi/3$

→ $\int_{\pi/3}^{\pi/2}\int_0^1 (r\cos\theta)\,r\,dr\,d\theta =$ $\boxed{\frac{1}{3}\left(1-\frac{\sqrt{3}}{2}\right)}$

---

⑲ $f(x,y) = x-y$; $x^2+y^2 \le 1$, $x+y \ge 1$

→ $\int_0^{\pi/2}\int_{\frac{1}{\cos\theta+\sin\theta}}^{1} r(\cos\theta-\sin\theta)\,r\,dr\,d\theta =$ $\boxed{0}$

㉗ $f(x,y,z) = x^2+y^2$; $x^2+y^2 \le 9$, $0 \le z \le 5$

→ $\int_0^{5}\int_0^{3}\int_0^{2\pi} r^2\,dr\,d\theta\,dz =$ $\boxed{\frac{405\pi}{2}}$

㉛ $f(x,y,z) = z$; $x^2+y^2 \le z \le 9$

$\rightarrow \int_0^3 \int_0^{2\pi} \int_{r^2}^{9} z\,dz\,d\theta\,dr = \boxed{243\pi}$

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(47) $f(x,y,z) = x^2+y^2$; $\rho \le 1$

$\rightarrow \int_0^{2\pi} \int_0^{\pi} \int_0^{1} (\rho^2 \sin^2\phi)\,\rho^2 \sin\phi\,d\rho\,d\phi\,d\theta = \boxed{\frac{8\pi}{15}}$

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(51) $f(x,y,z) = z$; $0 \le \theta \le \frac{\pi}{3}$, $0 \le \phi \le \frac{\pi}{2}$, $1 \le \rho \le 2$

$\rightarrow \int_0^{\pi/3} \int_0^{\pi/2} \int_1^{2} \rho^3 \cos\phi \sin\phi\,d\rho\,d\phi\,d\theta = \boxed{\frac{5\pi}{8}}$
