

HW 15.3-15.4

due 11/1/20

15.3

3. $f(x, y, z) = xe^{y-2z}$; $0 \leq x \leq 2$, $0 \leq y \leq 1$, $0 \leq z \leq 1$

$$\int_0^1 \int_0^1 \int_0^2 xe^{y-2z} dx dy dz \rightarrow \int_0^1 2e^{-2z}(e-1) dz$$

$$\boxed{(e^3 - e^2 - e + 1) / e^2}$$

5. $f(x, y, z) = (x-y)(y-z)$; $[0, 1] \times [0, 3] \times [0, 3]$

$$\int_0^3 \int_0^3 \int_0^3 (x-y)(y-z) dx dy dz \rightarrow \int_0^3 (-3z/2 + 9/4 + az/2 - 9) dz$$

$$\boxed{-z^2/4}$$

7. $f(x, y, z) = (x+z)^3$; $[0, a] \times [0, b] \times [0, c]$

$$\int_0^c \int_0^b \int_0^a (x+z)^3 dx dy dz = \int_0^c \frac{b}{4} [(a+z)^4 - z^4] dz \rightarrow \int_0^c \frac{b}{4} (a+z)^4 - z^4 dz$$

$$\boxed{\frac{b}{20} [(a+c)^5 - a^5 - c^5]}$$

9. $f(x, y, z) = x+y; W: y \leq z \leq x, 0 \leq y \leq x, 0 \leq x \leq 1$

$$\int_0^1 \int_0^x \int_y^x (x+y) dz dy dx \rightarrow \int_0^1 (x^2 - \frac{x^3}{3}) dx \rightarrow \boxed{\frac{1}{6}}$$

11. $f(x, y, z) = xyz; W: 0 \leq z \leq 1, 0 \leq y \leq \sqrt{1-x^2}, 0 \leq x \leq 1$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^1 xyz dy dx dz \rightarrow \int_0^1 \frac{z}{8} dz \rightarrow \boxed{1/16}$$

13. $f(x, y, z) = e^z; W: x+y+z \leq 1, x \geq 0, y \geq 0, z \geq 0$

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} e^z dz dy dx \rightarrow \int_0^1 e^{1-x} + x - 1 dx \rightarrow \boxed{e - 5/2}$$

15. $f(x, y, z) = z$

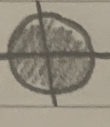
$$\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho \cos \theta d\rho d\theta d\phi \rightarrow \int_0^{\pi/2} \int_0^{\pi/2} \frac{1}{2} \cos \theta d\theta d\phi \rightarrow \boxed{\frac{1}{6}}$$

17. $f(x, y, z) = x$ above $z = y^2$ below $z = 8 - 2x^2 - y^2$

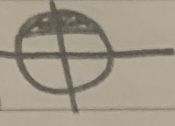
$$\int_0^2 \int_0^{\sqrt{8-2x^2}} \int_{y^2}^{8-2x^2-y^2} \rho \sin \theta \cos \theta d\rho d\theta d\phi \rightarrow \int_0^2 \int_0^{\sqrt{8-2x^2}} \frac{1}{2} \sin \theta \cos \theta d\theta d\phi \rightarrow \boxed{\frac{128}{15}}$$

15.4

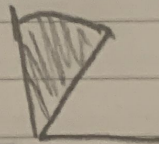
1. $f(x, y) = \sqrt{x^2 + y^2}, x^2 + y^2 \leq 2$

$$\int_0^{2\pi} \int_0^{\sqrt{2}} \sqrt{x^2 + y^2} dA \rightarrow \int_0^{2\pi} \int_0^{\sqrt{2}} r^2 dr d\theta \rightarrow \boxed{\frac{4\sqrt{2}\pi}{3}}$$


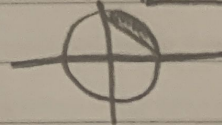
5. $f(x, y) = y(x^2 + y^2)^{-1}; y \geq 1/2, x^2 + y^2 \leq 1$

$$\int_0^{2\pi} \int_{1/2}^1 y(x^2 + y^2)^{-1} dA \rightarrow \int_0^{2\pi} \int_{1/2}^1 r \sin \theta dr d\theta \rightarrow \boxed{\sqrt{3} - \pi/3}$$


9. $\int_0^{1/2} \int_{\sqrt{3x}}^{\sqrt{1-x^2}} x dy dx \rightarrow \int_0^{\pi/2} \int_0^1 r^2 \cos \theta dr d\theta \rightarrow \boxed{\frac{1}{3} - \frac{\sqrt{3}}{6}}$



19. $f(x, y) = x-y; x^2 + y^2 \leq 1, x+y \geq 1$

$$\int_{\pi/2}^{\pi} \int_{1/2}^1 (r \cos \theta - r \sin \theta) dr d\theta \rightarrow \boxed{0}$$


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$$x = \rho \sin \theta \cos \phi \quad y = \rho \sin \theta \sin \phi$$

27. $\int (x, y, z) = x^2 + y^2; x^2 + y^2 \leq 9, 0 \leq z \leq 5$
 $\int_0^{2\pi} \int_0^{\pi} \int_0^3 \rho^2 d\rho d\theta d\phi = \frac{405\pi}{2}$

31. $\int (x, y, z) = z; x^2 + y^2 \leq z \leq 9$
 * review (confused)

47. $f(x, y, z) = x^2 + y^2; \rho \leq 1$
 $\int \int \rho^2 \sin^2 \theta \cos^2 \phi + \rho^2 \sin^2 \theta \sin^2 \phi d\rho d\theta d\phi$
 $= \frac{8\pi}{15}$

51. $f(x, y, z) = z; 0 \leq \theta \leq \frac{\pi}{3}, 0 \leq \phi \leq \frac{\pi}{2}, 1 \leq \rho \leq 2$
 $\int_0^{\pi/2} \int_0^{\pi/3} \int_1^2 \rho \cos \theta d\rho d\theta d\phi$
 $= \frac{\pi^2}{8}$