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Exercise 17.1

Q1. $\because x = r\cos\theta \quad P = xy$

$y = r\sin\theta \quad Q = y.$

this is unit circle.

$\therefore \theta = 0..2\pi$

$x^2 + y^2 = 1$

$y = \pm\sqrt{1-x^2}$

$\therefore -\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$

$-1 \le x \le 1$

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (0-x)\,dy\,dx$$

$$= 0 = \iint_D \left(\frac{dy}{dx} - \frac{d(xy)}{dy}\right) dA$$

$$= \frac{dP}{dy} - \frac{dQ}{dx}$$

$$\therefore \int_0^{2\pi} -(\cos\theta\sin\theta)\cdot(\sin\theta) + \sin\theta\cos\theta \quad d\theta$$

$= 0.$

Q3. $\oint_C y^2\,dx + x^2\,dy$

$$= \int_0^1\int_0^1 2x-2y\, \cancel{D-0}\, dx\,dy$$

$= 0$

Q5. $\oint_C x^2 y\,dx$

$$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (2xy - x^2)\,dy\,dx$$

$= -\frac{\pi}{4}$

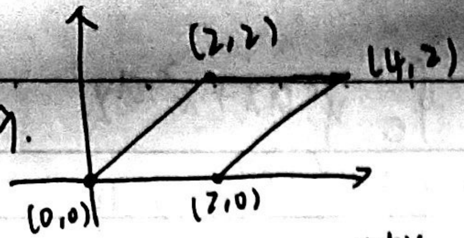
Q7. $F(x,y) = (x^2, x^2)$

$y = x^2 \quad y = x$

$$\int_0^1\int_x^{x^2} (2x-0)\,dy\,dx$$

$$= \cancel{\int_0^1 2x^3 - 2x^2\, dx}$$

$= -\frac{1}{6}$



Q9.

$$F(x,y) = (e^{x+y}, e^{x-y})$$

$$P_y = e^{x+y}$$

$$Q_x = e^{x-y}$$

~~from 0 to 2 (x)~~

~~$x=2t,\ y=2t$~~

$$\int_0^2 \int_0^x e^{x-y} - e^{x+y}\, dy\, dx$$

$$+ \int_2^4 \int_{x-2}^2 (e^{x-y} - e^{x+y})\, dy\, dx$$

$$= -\frac{5}{2} + \frac{5e^2}{2} + \frac{e^4}{2} - \frac{e^6}{2}$$

$$= 158.44$$

$$-158.44 \times -1 = 158.44$$

Q13. $P = \sin x + y$

$Q = 3x + y$

$Q_x = 3$

$P_y = 1$

$\therefore Q_x - P_y = 2$

$$\int_0^2 \int_2^2 dy\, dx \ \cancel{dx\, dy}$$

$$\cancel{= 2 \cdot \text{area}} + \int_0^2 \int_2^4 2\, dy\, dx$$

$$+ \int_0^2 \int_4^{6-\frac{x}{2}} dy\, dx$$

$$+ \int_0^6 y\, dy$$

$$= 34.$$

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Exercise 17.2

Q1. $x^2+y^2+z=1$

$x^2+y^2\leq 1$

∴ $r=1$

$x=\cos t$, $y=\sin t$ $z=0$

∴ $F=2(\cos t)(\sin t)i+\cos t j$
$+\sin t k$

$r(t)=\cos t\, i+\sin t j+0k$

$F\cdot dr=\cancel{2\cos^2+\sin t+\sin t\cos t}$
$-2\cos\theta\sin^2\theta+\cos^2\theta+0\, d\theta$

$=\int_0^{2\pi} F\cdot dr$

$=\pi$

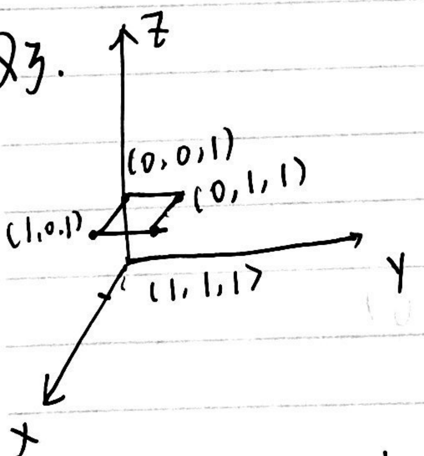
$\text{curl}(F)=(1,0,1-2y)$

∴ $\iint (-2x)\cdot(-1)-0+(1-2x)\ dS$

$=\iint 1\, dS$

$=\pi$

Q3.



$F=(e^{y-z},0,0)$

$$\text{curl}(F)=\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ e^{y-z} & 0 & 0 \end{vmatrix}$$

$=i0+(-e^{y-z})j+(-e^{y-z})k$

$=(0,-e^{y-z},-e^{y-z})$

$\cancel{r-x=}$

$\iint_S \text{curl}(F)\cdot dS$

$=\int_0^1\int_0^1 -e^{y-1}\ dx\,dy$

$=e^{-1}-1$

Q5. $\operatorname{curl}(F) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{z^2}-y & e^{z^3}+x & \cos(xz) \end{vmatrix}$ $\cancel{x^2+y^2+z^2=1}$

$z \geq 0.$

$= i(0 - 3z^2 \cdot e^{z^3}) - j(z\cdot\cos(xz) - 2z\cdot e^{z^2})$

$+ k(1-(-1))$

$= (-3z^2\cdot e^{z^3}, + z\cdot \overset{\sin}{\cancel{\cos}}(xz) - 2z\cdot e^{z^2}, 2)$

$x=\cos t \quad y=\sin t \quad z=0$

$r(t) = \cos t\, i + \sin t\, j + 0k$

$r'(t) = -\sin t\, i + \cos t\, j$

$F = (e^{z^2}-y, e^{z^3}+x, \cos xz)$

$=(1-y, 1+x, \cos 0)$

$=(1-\sin t, 1+\cos t\, j, 1)$

$\int_0^{2\pi} F\cdot dr = \int_0^{2\pi} \cos t - \sin t + 1$

$= 2\pi.$

Q9. $\operatorname{curl} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} = (0,0,0)$

$\therefore$ it is a conservative vector field.

$\therefore$ the answer is 0.

Campus

$F=(3y, -2x, 3y) \quad x^2+y^2=9 \quad z=2$

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$$\text{Q11. } \operatorname{curl}(F) = \begin{array}{c} \begin{array}{ccc} i & j & k \end{array} \\ \left| \begin{array}{ccc} \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 3y & -2x & 3y \end{array} \right| \end{array} = (3-0)i - j(0-0) + k(-2-3)$$

$$= (3, 0, -5)$$

$$= \cancel{0} -2.$$

$x=3\cos t \quad y=3\sin t \quad z=2$

$F=(3 \cdot 3\sin t, -2(3\cos t), 3(3\sin t))$

$dr=(-3\sin t, 3\cos t, 0)$

$$\cancel{\int_0^{3\pi} F \cdot dr = 9\sin t \frac{3\pi}{2} \quad 2)} = -2$$

(I'm not sure with this).

Q13. $\because \vec{PQ} = (0,3,3)$

$\vec{PR} = (3,0,0)$

$\vec{PQ} \times \vec{PR} = (0,9,-9)$

$\therefore z = y$

$\text{curl } F = (-1,-1,-1)$

$\therefore \int_0^3 \int_0^x 0 \, dA = 0.$