Math 251 Shaun Boda Section 23 HW#12

January February March April May June July August September October November December

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

17.1:

1) $\oint_C xy\,dx + y\,dy = -\iint_D \left(\overset{0}{\cancel{x}} - \overset{x}{\cancel{y}}\right) dx\,dy$

for circle

$$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (x)\,dy\,dx = \iint_D \left(\frac{\partial}{\partial x} y - \frac{\partial}{\partial y}(xy)\right) dA$$

3) $\oint_C y^2\,dx + x^2\,dy = -\iint_D (2x-2y)\,dx\,dy = -\int_0^1\int_0^1 (2x-2y)\,dx\,dy$

$$\Rightarrow \int_0^1 (2x-2y)\,dx = \left| x^2 - 2yx \right|_0^1 = 0$$

$$\int_0^1 0\,dy = \boxed{0}$$

5) $\oint_C x^2y\,dx = \oint_C x^2y\,dx + 0\,dy = -\iint_D (0-x^2)\,dx\,dy$

$$= -\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (-x^2)\,dy\,dx \Rightarrow \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (-x^2)\,dy = \left| -x^2y \right|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}$$

$$= -2x^2\sqrt{1-x^2} \Rightarrow -\int_{-1}^{1} \left(-2x^2\sqrt{1-x^2}\right) dx = \boxed{-\frac{\pi}{4}}$$

7) $\oint_C F\cdot dr = \oint_C x^2\,dx + x^2\,dy = -\iint_D (2x-0)\,dy\,dx = \int_0^1\int_{x^2}^{x} (2x)\,dy\,dx$

$$= \int_{x^2}^{x} 2x\,dy = \left|2xy\right|_{x^2}^{x} = 2x^2 - 2x^3$$

$$-\int_0^1 (2x^2-2x^3)\,dx = -\left|\frac{2x^3}{3} - \frac{x^4}{2}\right|_0^1 = -\left(\frac{4}{6} - \frac{3}{6}\right)$$

$$= \boxed{\frac{1}{6}}$$

9) $\oint_C F\,dr = \oint_{\partial C} e^{x+y}\,dx + e^{x-y}\,dy = \iint_D \left(e^{x-y} - e^{x+y}\right)dx\,dy$

$= \int_0^2 \int_0^2 \left(e^{x-y} - e^{x+y}\right) dx\,dy$

$= \int_0^2 \left(e^{x-y} - e^{x+y}\right) dx = \left| -2e^{x} \sinh(y) \right|_0^2$

$= \int_0^2 \text{ans}\, dy \approx \boxed{-35.29}$

13) $\int_C (\sin x + y)\,dx + (3x + y)\,dy = \iint_D (3 - \cos x + y)\,dx\,dy$

$\int_0^2 \int_0^2 (3 - \cos(x+y))\,dx\,dy = \cos(4) - 2\cos(2) + 13$

$\int_2^4 \int_2^2 (3 - \cos(x+y))\,dx\,dy = 0$

$\int_4^6 \int_2^0 (3 - \cos x + y)\,dx\,dy = -\cos(8) + 2\cos(6) - \cos(4) - 12$

$\int_6^0 \int_0^0 (3 - \cos x + y)\,dx\,dy = 0$

Sum $= \boxed{-\cos(8) + 2\cos(6) - 2\cos(2) + 1 \approx 3.848}$

17.2:

1) $r(t) = \cos t\, i + \sin t\, j + k$ for $0 \le t \le 2\pi$

$F(r(t)) = \langle 2\cos(t)\sin(t), \cos t, \sin t + 1 \rangle$

$r'(t) = -\sin(t) i + \cos(t) j + 0k$

$F(r(t)) \cdot r'(t) = -2\cos t \sin^2 t + \cos^2 t$

$\int_0^{2\pi} (-2\cos t \sin^2 t + \cos^2 t)\,dt = \boxed{\pi}$

3) $\oint_0^1 \oint_0^1 F\, dS = \int \text{curl } F$

$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{y-z} & 0 & 0 \end{vmatrix} = 0i - (-e^{y-z}) j + (-e^{y-z}) k$

$\text{curl } F = \langle 0, e^{y-z}, e^{y-z} \rangle \qquad z = 1$

$\oint F \cdot dr = \iint_D \langle 0, e^{y-z}, e^{y-z} \rangle \cdot \langle 0, 0, 1 \rangle\, dA$

$= \int_0^1 \int_0^1 e^{y-z}\, dy dx \approx \boxed{1.086}$

5) $\text{curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{z^2} - y & e^{z^3} + x & \cos(xz) \end{vmatrix}$ $\qquad g(x) = \sqrt{1 - x^2 - y^2}$ $\qquad g'(x) = \frac{-x}{\sqrt{-x^2 - y^2 + 1}}$

$= (0 - 3e^{z^3}) i - (-z\sin(xz) - 2e^{z^2}) j + (1+1) k$

$= \langle -3e^{z^3}, z\sin(xz) + 2e^{z^2}, 2 \rangle$

$\text{flux} = \iint_S \langle -3e^{z^3}, z\sin(xz) + 2e^{z^2}, 2 \rangle \cdot \langle \frac{x}{\sqrt{-x^2-y^2+1}}, \frac{y}{\sqrt{-x^2-y^2+1}}, 1 \rangle\, dA$

$= \boxed{2\pi}$

9) $\text{curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} = (x-x)i - (y-y)j + (z-z)k$
$= \langle 0, 0, 0 \rangle$

answer is $\boxed{0}$

11) $\text{curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3y & -2x & 3y \end{vmatrix} = (3-0)i - (0-0)j + (-2-3)k$
$= \langle 3, 0, -5 \rangle$

flux of curl F $= \iint_S (3x - 5\cancel{z}^{(2)})\, dS$

$= \int_0^3 \int_0^{2\pi} (3x - 10)\, d\theta\, dr = \boxed{-45\pi}$

$x \to r\cos\theta$

13) $\text{curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z & x \end{vmatrix} = (0-1)i - (1-0)j + (0-1)k$
$= \langle -1, -1, -1 \rangle$

flux of curl F $= \iint_S (-x-y-z)\, dS$

$= \int_0^3 \int_0^3 \int_0^3 (-x-y-z)\, dx\, dy\, dz = \boxed{-\frac{243}{2}}$