HW 12/6/20

17.1: 1, 3, 5, 7, 9, 13

17.2: 1, 3, 5, 9, 11, 13

Sec 17.1:

1. $\int_C xy\,dx + y\,dy$

$x = \cos\theta \quad y = \sin\theta \quad 0 \le t \le 2\pi$

$dx = -\sin\theta\,d\theta \quad dy = \cos\theta\,d\theta$

$xy\,dx + y\,dy = (-\sin^2\theta\cos\theta + \sin\theta\cos\theta)\,d\theta$

$\int_0^{2\pi}(-\sin^2\theta\cos\theta + \sin\theta\cos\theta)\,d\theta$

$= -\int_0^{2\pi}\sin^2\theta\cos\theta\,d\theta + \frac{1}{2}\int_0^{2\pi} 2\sin\theta\cos\theta\,d\theta$

$= \frac{-\sin^3\theta}{3}\Big|_0^{2\pi} - \frac{\cos 4\theta}{4}\Big|_0^{2\pi}$

$= -\frac{1}{3}(\sin^3(2\pi) - \sin^3(0)) - \frac{1}{4}(\cos(8\pi) - \cos(0))$

$= -\frac{1}{3}(0-0) - \frac{1}{4}(1-1) = \boxed{0}$

$P = xy \quad Q = y$

$\frac{dP}{dy} = x \quad \frac{dQ}{dx} = 0 \rightarrow \frac{dQ}{dx} - \frac{dP}{dy} = 0 - x = -x$

$x^2 + y^2 \le 1 \quad \iint_D (-x)\,dx\,dy = -\iint_D x\,dx\,dy = \boxed{0}$

3. $F = \langle P, Q\rangle = \langle y^2, x^2\rangle$

$\text{curl}_z(F) = \frac{dQ}{dx} - \frac{dP}{dy} = \frac{d}{dx}(x^2) - \frac{d}{dy}(y^2) = 2x - 2y$

$0 \le x \le 1 \quad 0 \le y \le 1$

$\iint_D (2x-2y)\,dA = \int_0^1\int_0^1 (2x-2y)\,dx\,dy$

$= x^2 - 2xy\Big|_0^1 = 1 - 2y$

$= y - y^2\Big|_0^1 = (1-1) - (0-0) = \boxed{0}$

5. $P = x^2 y \quad Q = 0$

$\frac{dP}{dy} = x^2 \quad \frac{dQ}{dx} = 0$

$-\iint_D x^2\,dA \rightarrow -\int_0^{2\pi}\int_0^1 r^3\left(\frac{1+\cos 2\theta}{2}\right)dr\,d\theta$

$-\frac{1}{4}\int_0^{2\pi}\left(\frac{1+\cos 2\theta}{2}\right)d\theta = \theta + \frac{\sin 2\theta}{2}\Big|_0^{2\pi}$

$= -\frac{1}{8}(2\pi + 0) = \boxed{-\frac{\pi}{4}}$

7. $P = x^2 \quad Q = x^2$

$\frac{dP}{dy} = 0 \quad \frac{dQ}{dx} = 2x \quad 0 \le x \le 1$

$\iint_D 2x\,dA = \int_0^1\int_{x^2}^{x} 2x\,dy\,dx$

$= 2xy\Big|_{x^2}^{x} = 2x^2 - 2x^3$

$\int_0^1 2x^2 - 2x^3\,dx = \frac{2x^3}{3} - \frac{x^4}{2}\Big|_0^1 = \left(\frac{2}{3} - \frac{1}{2}\right) - 0$

$= \boxed{\frac{1}{6}}$

9. $F = \langle e^{x+y}, e^{x-y}\rangle$

$\frac{dP}{dy} = e^{x+y} \quad \frac{dQ}{dx} = e^{x-y}$

$-\iint_D (e^{x-y} - e^{x+y})\,dA$

$= -\int_0^2\int_0^x (e^{x-y} - e^{x+y})\,dy\,dx - \int_2^4\int_{x-2}^{2}(e^{x-y} - e^{x+y})\,dy\,dx$

$= (-e^{x-y} - e^{x+y})\Big|_0^x - (-e^{x-y} - e^{x+y})\Big|_{x-2}^{2}$

$= \int_0^2 (-1 + 2e^x - e^{2x})\,dx - \int_2^4 (-e^{x-2} + e^2 - e^{x+2} + e^{2x-2})\,dx$

$= -(-x + 2e^x - \frac{1}{2}e^{2x})\Big|_0^2 - (-e^{x-2} + xe^2 - e^{x+2} + \frac{1}{2}e^{2x-2})\Big|_2^4$

$= \boxed{\frac{5}{2} - \frac{1}{2}e^4 + \frac{1}{2}e^6 - \frac{5}{2}e^2}$

13. $P = \sin x + y \quad Q = 3x + y$

$\frac{dP}{dy} = 1 \quad \frac{dQ}{dx} = 3$

$\iint_D (3-1)\,dA = 2\iint_D dA = 2\left(\frac{1}{2}\right)(6+2)(2)$

$= \boxed{16}$

Sec 17.2:

1. $F = \langle 2xy, x, y+z\rangle \quad z = 1 - x^2 - y^2 \quad x^2 + y^2 \le 1$

$C(t) = (\cos t, \sin t, 0) \quad 0 \le t \le 2\pi$

$C'(t) = (-\sin t, \cos t, 0)$

$F(C(t)) = \langle 2\cos t\sin t, \cos t, \sin t\rangle$

$\langle 2\cos t\sin t, \cos t, \sin t\rangle \cdot \langle -\sin t, \cos t, 0\rangle$

$= -2\cos t\sin^2 t + \cos^2 t$

$= \int_0^{2\pi}(-2\sin^2 t\cos t + \cos^2 t)\,dt =$

$= \left(-\frac{2}{3}\sin^3 t + \frac{t}{2} + \frac{\sin 2t}{4}\right)\Big|_0^{2\pi}$

$= \left(-\frac{2}{3}\sin^3(2\pi) + \frac{2\pi}{2} + \frac{\sin 4\pi}{4}\right) - (0)$

$= \boxed{\pi}$

Sec 17.2 (cont.)

1. (cont.)

$\Phi(\theta,t) = (t\cos\theta, t\sin\theta, 1-t^2) \quad 0 \le t \le 1 \quad 0 \le \theta \le 2\pi$

$T_\theta = \langle -t\sin\theta, t\cos\theta, 0\rangle \quad T_t = \langle \cos\theta, \sin\theta, -2t\rangle$

$T_\theta \times T_t = \begin{vmatrix} i & j & k \\ -t\sin\theta & t\cos\theta & 0 \\ \cos\theta & \sin\theta & -2t \end{vmatrix} = -2t^2\cos\theta\, i - 2t^2\sin\theta\, j - t k$

$n = \langle 2t^2\cos\theta, 2t^2\sin\theta, t\rangle$

$\mathrm{curl}(F) = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ 2xy & x & y+z \end{vmatrix} = i + (1-2x)k = \langle 1, 0, 1-2x\rangle$

$\mathrm{curl}(F) = \langle 1, 0, 1-2t\cos\theta\rangle$

$\mathrm{curl}(F)\cdot n = \langle 1, 0, 1-2t\cos\theta\rangle \cdot \langle 2t^2\cos\theta, 2t^2\sin\theta, t\rangle$

$= 2t^2\cos\theta + 0 + t - 2t^2\cos\theta = t$

$\int_0^{2\pi}\int_0^1 t\, dt\, d\theta = \frac{t^2}{2}\Big|_0^1 = \frac{1}{2}$

$= \frac{1}{2}\theta\Big|_0^{2\pi} = \boxed{\pi}$

3. $F(x,y,z) = \langle e^{y-z}, 0, 0\rangle$

$\mathrm{curl}(F) = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ e^{y-z} & 0 & 0 \end{vmatrix} = -e^{y-z} j - e^{y-z} k$

$\mathrm{curl}(F)\cdot n = (-e^{y-z} j - e^{y-z} k)\cdot k = -e^{y-z}$

$\iint_S \mathrm{curl}(F)\cdot n\, dS = \boxed{e^{-1} - 1}$

5. $F = \langle e^{z^2} - y, e^{z^3} + x, \cos(xz)\rangle$

$x^2 + y^2 + z^2 = 1 \quad z \ge 0$

$\mathrm{curl}(F) = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ e^{z^2} - y & e^{z^3} + x & \cos(xz) \end{vmatrix}$

$= \langle -3z^2 e^{z^3}, z\sin(xz) + 2z e^{z^2}, 2\rangle$

$c(t) = \langle \cos t, \sin t, 0\rangle \quad 0 \le t \le 2\pi$

$c'(t) = \langle -\sin t, \cos t, 0\rangle$

$F(c(t)) = \langle 1 - \sin t, 1 + \cos t, 1\rangle$

$F(c(t))\cdot c'(t) = \langle 1 - \sin t, 1 + \cos t, 1\rangle \cdot \langle -\sin t, \cos t, 0\rangle$

$= -\sin t + \sin^2 t + \cos t + \cos^2 t + 0 = 1 - \sin t + \cos t$

$\int_0^{2\pi} (1 - \sin t + \cos t)\, dt = t - \cos t - \sin t \Big|_0^{2\pi}$

$= (2\pi - 1 - 0) - (0 - 1 - 0)$

$= \boxed{2\pi}$

9. $F = \langle yz, xz, xy\rangle \quad x^2 + y^2 = 1 \quad z = 1 \quad z = 4$

$\mathrm{curl}(F) = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ yz & xz & xy \end{vmatrix}$

$= (x-x)i - (y-y)j + (z-z)k = \langle 0, 0, 0\rangle$

$= \boxed{0}$

11. $F = \langle 3y, -2x, 3y\rangle \quad x^2 + y^2 = 9 \quad z = 2$

$\mathrm{curl}(F) = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ 3y & -2x & 3y \end{vmatrix}$

$= i(3-0) - j(0-0) + k(-2-3)$

$= \langle 3, 0, -5\rangle$

$r(x,y) = \langle x, y, 2\rangle \quad 0 \le x \le 3 \quad 0 \le y \le 3$

$r_x = \langle 1, 0, 0\rangle \quad r_y = \langle 0, 1, 0\rangle$

$r_x \times r_y = \langle 0, 0, 1\rangle$

$\mathrm{curl}(F)\cdot\langle 0, 0, 1\rangle = 0 + 0 + (-5)(1) = -5$

$\mathrm{Flux} = \int_0^3\int_0^3 -5\, dy\, dx = -5y\Big|_0^3 = -15$

$-15x\Big|_0^3 = \boxed{-45}$

13. $F = \langle y, z, x\rangle \quad (0,0,0)\ (3,0,0)\ (0,3,3)$

$\mathrm{curl}(F) = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ y & z & x \end{vmatrix}$

$= i(0-1) - j(1-0) + k(0-1) = \langle -1, -1, -1\rangle$

$r(x,y) = \langle x, y, z\rangle \quad 0 \le x \le 3 \quad 0 \le y \le 3 \quad 0 \le z \le 3$

$r_x = \langle 1, 0, 0\rangle \quad r_y = \langle 0, 1, 0\rangle$

$r_x \times r_y = \langle 0, 0, 1\rangle$

$\mathrm{curl}(F)\cdot\langle 0, 0, 1\rangle = 0 + 0 + (-1)(1) = -1$

$\int_0^3\int_0^3 -1\, dy\, dx = -1y\Big|_0^3 = -3$

$-3x\Big|_0^3 = \boxed{-9}$