Calc HW
Due 12/6

Rahul Paleja

17.1 D #1, 3, 5, 7, 9, 13:

(1) verify Green's Theorem for line integral $\oint_C xy\,dx + y\,dy$ where C is the unit circle, oriented counterclockwise

$x = \cos\theta,\ y = \sin\theta;\ 0 \le \theta \le 2\pi$
$dx = -\sin\theta\,d\theta,\ dy = \cos\theta\,d\theta$

$$\int_0^{2\pi} (\cos\theta\sin\theta)(-\sin\theta) + (\sin\theta)(\cos\theta)\,d\theta$$

$$= \int_0^{2\pi} -\cos\theta\sin^2\theta + \sin\theta\cos\theta\,d\theta$$

$u = \sin\theta \quad du = \cos\theta$ $\qquad u = \cos\theta \quad \frac{du}{-1} = \frac{-\sin\theta\,d\theta}{-1}$

$$-\frac{\sin^3\theta}{3}\Big]_0^{2\pi} - \frac{\cos^2\theta}{2}\Big]_0^{2\pi} = 0 - 0 = \boxed{0}$$

$P = xy \qquad Q = y$

$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0 - x = -x$

Green's Theorem:

$$-\iint_D x\,dx\,dy = \boxed{0}$$

since $\oint_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$

(3) $\oint y^2\,dx + x^2\,dy$ where C is the boundary of the unit square with $0 \le x \le 1, 0 \le y \le 1$

$P = y^2 \quad Q = x^2$

$$\int_0^1\int_0^1 x^2\,dx - y^2\,dy = \int_0^1\int_0^1 2x - 2y$$

Inner:

$$2\int_0^1 x - y\,dx = \frac{x^2}{2} - xy\Big]_0^1 = \frac{1}{2} - y$$

Outer:

$$2\int_0^1 \frac{1}{2} - y\,dy = 2\left(\frac{1}{2}y - \frac{y^2}{2}\right]_0^1 = 2\left(\tfrac{1}{2} - \tfrac{1}{2}\right) - (0 - 0) = \boxed{0}$$

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⑤ $\oint_C x^2 y\,dx$ where C is the unit circle centered at the origin

$P = x^2 y \quad Q = 0$

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy = \iint_D -x^2\,dx\,dy$$

convert to polar $x = r\cos\theta$

$$\int_0^{2\pi}\int_0^1 -(r\cos\theta)^2\, r\,dr\,d\theta = -\int_0^{2\pi}\int_0^1 r^3\cos^2\theta\,dr\,d\theta$$

inner:

$$-\cos^2\theta \int_0^1 r^3\,dr = \frac{r^4}{4}\Big]_0^1 = \frac{1}{4}$$

<u>Outer:</u>

$$-\frac{1}{4}\int_0^{2\pi}\cos^2\theta\,d\theta = -\frac{1}{4}\left(\frac{1}{2}\theta + \frac{\sin 2\theta}{4}\right]_0^{2\pi}$$

$$= \boxed{\frac{-\pi}{4}}$$

⑦ $\oint_C F\,dr$ where $F(x,y) = (x^2, x^2)$ & C consists of the arcs $y = x^2$ and $y = x$ for $0 \le x \le 1$

$P, Q = x^2 \qquad \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2x - 0 = 2x$

$$\int_0^1\int_{x^2}^{x} 2x\,dy\,dx$$

Inner: $2x\,y\Big]_{x^2}^{x} = 2x(x - x^2) = 2x^2 - 2x^3$

$$\int_0^1 2x^2 - 2x^3\,dx = \frac{2x^3}{3}\Big]_0^1 - 2\frac{x^4}{4}\Big]_0^1$$

$$= \left(\frac{4}{4}\right)\frac{2}{3} - \frac{2}{4}\left(\frac{3}{3}\right) = \frac{2}{12} = \boxed{\frac{1}{6}}$$

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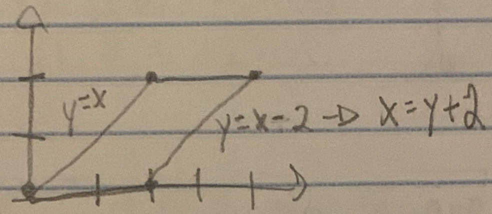
17.1 → 9, 13:

(9) The line integral of $F(x,y) = \langle e^{x+y}, e^{x-y} \rangle$ along curve (oriented clockwise) consisting of line segments by joining pts. (0,0), (2,2), (4,2), (2,0) and back to (0,0)



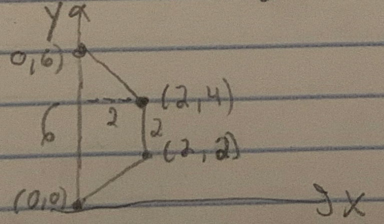
$P = e^{x+y}$ $Q = e^{x-y}$

$\frac{dQ}{dx} = e^{x-y}$ $\frac{dP}{dy} = e^{x+y}$

$$\int_0^2 \int_y^{y+2} (e^{x-y}) - (e^{x+y}) \, dx\,dy = (e^2 - 1)\left(\frac{5 - e^4}{2}\right)$$ → Done On Maple

(13) Evaluate $I = \int_C (\sin x + y)dx + (3x + y)dy$ for:

↑ Ask In Recitation



$P = \sin x + y$ $Q = 3x + y$

$\frac{dQ}{dx} = 3 - \frac{dP}{dy} = 1$

$= 2$

Area Of Trapezoid: $\frac{6+2}{2}(2) = 8(2) = \boxed{16}$

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17.2 → #1, 3, 5, 9, 11, 13:

① $F = \langle 2xy, x, y+z \rangle$, surface $z = 1 - x^2 - y^2$ for $x^2 + y^2 \leq 1$

Must prove $\iint_C F ds = \iint_S \text{curl}(F) ds$

$\text{curl}(F) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy & x & y+z \end{vmatrix}$

$= i(1) - j(0) + k(1-2x)$

$= \langle 1, 0, 1-2x \rangle$

$\int_0^1 \int_0^{2\pi} i + (1-2x)\, r\,dr\,d\theta = \boxed{\pi}$

$= \iint_C F \cdot ds$

③ $F = \langle e^{y-z}, 0, 0 \rangle$, the square with vertices $(1,0,1)$, $(1,1,1)$, $(0,1,1)$, and $(0,0,1)$

$\iint_C F \cdot ds = \iint_S \text{curl}(F) \cdot ds$

$\text{curl}(F) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{y-z} & 0 & 0 \end{vmatrix}$

$i(0) - j(0 + e^{y-z}) + k(-e^{y-z})$

$= \langle 0, -e^{-y-z}, -e^{y-z} \rangle$

$\iint_C e^{y-z}\, dx\,dy = \iint_S \langle 0, -e^{y-z}, -e^{y-z} \rangle ds$

$= \boxed{e^{-1} - 1}$

⑤ $F = \langle e^{z^2} - y, e^{z^3} + x, \cos(xz) \rangle$, the upper hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$ with outward pointing normal

$\iint_S F ds = \iint_S \text{curl}(F) ds$

$\text{curl}(F): \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{z^2} - y & e^{z^3} + x & \cos(xz) \end{vmatrix}$

$= i(0 - 3z^2 e^{z^3}) - j(-\sin(xz) - 2ze^{z^2} - y)$

$+ k(1 - 1)$

$= \langle -3z^2 e^{z^3}, -\sin(xz) \cdot 2ze^{z^2} - y, 0 \rangle$

$\iint_S \langle -3z^2 e^{z^3}, -\sin(xz) \cdot 2ze^{z^2} - y, 0 \rangle dS$

$= \langle -3z^2 e^{z^3}, 2ze^{z^2} + z\sin(xz), 2 \rangle$

9) $F = \langle yz, xz, xy \rangle$ that part of the cylinder $x^2+y^2=1$ that lies between the 2 planes $z=1$ and $z=4$ with outward-pointing unit normal vector

$\iint_C F \cdot ds = \iint_S \text{curl}(F)\, ds$

$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & xz & xy \end{vmatrix}$ $i(x-x) - j(y-y) + k(z-z) = \langle 0,0,0 \rangle$

$\text{curl}(F) = \langle 0,0,0 \rangle$ so integral must equal $\boxed{0}$

11) $F = \langle 3y, -2x, 3y \rangle$ C is the circle $x^2+y^2=9, z=2$, oriented counterclockwise as viewed from above.

$\iint_C F \cdot ds = \iint_S \text{curl}(F)\, ds$

$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3y & -2x & 3y \end{vmatrix} = i(3) - j(0) + k(-2-3) = \langle 3, 0, -5 \rangle$

$\int_0^{2\pi} \int_0^3 (3\cos\theta)^2 + (0)^2\, r\, dr\, d\theta = \boxed{-45\pi}$

13) $F = \langle y, z, x \rangle$
C is the triangle with vertices $(0,0,0)$, $(3,0,0)$ and $(0,3,3)$ oriented counterclockwise as viewed from above

$\text{curl}(F) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & z & x \end{vmatrix} = i(0-1) - j(1) + k(-1) = \langle -1, -1, -1 \rangle$

$\iint_C F\, ds = \iint_S \text{curl}(F)\, ds$

counterclockwise $\Rightarrow$ positive integral

$\iint_C \langle y, z, x \rangle\, ds = \iint_S \langle -1,-1,-1 \rangle\, dS$
$= \boxed{0}$