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17.1 : 1, 3, 5, 7, 9, 13

17.2 : 1, 3, 5, 9, 11, 13

① $\oint_C \overset{P}{xy}\,dx + \overset{Q}{y}\,dy$

$= \iint_R \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right] dA$

$= \iint_R [0 - x]\, dA$

$= \int_{-1}^{1} \left[\int_0^{2\pi} (\cos\theta\sin\theta(-\sin\theta) + \sin\theta\cos\theta)\, d\theta\right]$

go to the other quadrant too!

$\underbrace{\qquad\qquad}_{=0}$

$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (0 - x)\, dy\, dx = \iint_D \left(\frac{\partial}{\partial x} y - \frac{\partial}{\partial y}(xy)\right) dA$

③ $\int y^2 dx + x^2 dy \quad 0 \le x \le 1 \quad 0 \le y \le 1$

$= \int_0^1\int_0^1 [2x - 2y]\, dx\, dy$

$= \int_0^1 x^2 - 2xy = [1 - 2y]$

$= \int_0^1 (1 - 2y) = y - y^2 = 1 - 1 = \boxed{0}$

⑤ $\oint_C x^2 y\, dx$

$\int_{-1}^{1}\int_0^{2\pi} [-x^2]\, dy\, dx$

$\int_0^{2\pi} -x^2 dy = y - x^2 \Big|_0^{2\pi}$

$= -2\pi x^2 \longrightarrow \int_{-1}^{1} (-2\pi x^2)$

$= \frac{-2\pi x^3}{3}\Big|_{-1}^{1} = \frac{-2\pi}{3} + \frac{2\pi}{3}$

⑦ $\oint_C F\cdot dr \quad F(x,y) = \langle x^2, x^2\rangle$

$y = x^2$

$y = x$ upper

$0 \le x \le 1$

* always determine which bound is which! watch out!

$\int_0^1\int_{x^2}^{x} (2x)\, dy\, dx$

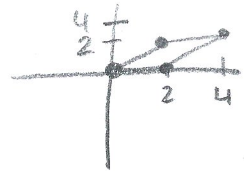
$= 2xy\Big|_{x^2}^{x} = 2x^2 - 2x^3$

$\int_0^1 (2x^2 - 2x^3)\, dx = \frac{2}{3}x^3 - \frac{x^4}{2}\Big|_0^1$

$= \frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \boxed{\frac{1}{6}}$

⑨ $F(x,y) = \langle \overset{P}{e^{x+y}}, \overset{Q}{e^{x-y}}\rangle$

$(0,0)(2,2)(4,2)(2,0)(0,0)$



$\iint_A (e^{x-y} - e^{x+y})\, dy\, dx$

$= \int_0^4\int_0^{2x} (e^{-2y})$

$= \left(-\frac{1}{2}\right)e^{-2y}\Big|_0^{2x} = -\frac{1}{2}e^{-4x} - \frac{1}{2}$

$\int_0^4 \left(-\frac{1}{2}e^{-4x} - \frac{1}{2}\right) = -\frac{e^{-4x}}{8} - \frac{x}{2}\Big|_0^4$

$= +\frac{1}{8} - \frac{e^{-16}}{8} - 2$

⑬ $I = \int_C (\sin x + y)\, dx + (3x + y)\, dy$

$\int_0^6 \int (2)$

17.2

① $F = \langle 2xy, x, y + z\rangle$

$z = 1 - x^2 - y^2$

$x^2 + y^2 \le 1$

$x = \cos t$

$y = \sin t$

$z = 0$

$F = \langle 2\cos t\sin t, \cos t, \sin t\rangle$

$dr = \langle -2\cos^2 t + 2\sin^2 t, -\sin t, \cos t\rangle$

$F\cdot dr = -4\cos^3 t\sin t + 2\sin^3 t\cos t, \cancel{-\cos t\sin t} + \cancel{\cos t\sin t}$

$\int_0^{2\pi} (-4\cos^3 t\sin t + 2\sin^3 t\cos t)\, dt$

$\boxed{\pi}$

③ $F = \langle e^{y-z}, 0, 0\rangle$

$(1,0,1)(1,1,1)$

$(0,1,1)(0,0,1)$

$= \iint_S \text{curl} F\cdot dS$

$\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{y-z} & 0 & 0 \end{vmatrix}$

$= (0-0)i - (e^{y-z})j + (-e^{y-z})k$

$\int_0^1\int_0^1$

(5) $F = \langle e^{z^2} - y,\ e^{z^3} + x,\ \cos(xz) \rangle$

turns to +

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{z^2}-y & e^{z^3}+x & \cos(xz) \end{vmatrix} = (0 - 3z^2 e^{z^3})i - (-z\sin(xz) - 2ze^{z^2})j + (1+1)k$$

curl $F = \langle -3z^2e^{z^3},\ z\sin(xz) - 2ze^{z^2},\ 2 \rangle$

$x^2+y^2+z^2 = 1 \quad z \geq 0$

$r(t) = \cos t\, i + \sin t\, j + 0k$

$r'(t) = -\sin t\, i + \cos t\, j$

$x = \cos t$

$y = \sin t$

$z = 0$

$F = \langle 0, 0, 2 \rangle$

(9) $F = \langle yz, xz, xy \rangle \quad x^2+y^2 = 1$

$z = 1 \quad z = 4$

curl $F = \langle 0, 0, 0 \rangle$

(so surface is irrelevant)

↳ since it is a closed curve and a conservative vector field (curl $F = \langle 0,0,0 \rangle$),

$\boxed{\int_C F \cdot dr = 0.}$

(11) $z = 2 \quad x^2+y^2 = 9 \quad z = 2$

$r = 3$ (disk, radius 3)

$$F = \langle \overset{P}{3y}, \overset{Q}{-2x}, \overset{R}{3y} \rangle$$

$x = 3\cos t$

$y = 3\sin t$

$z = 2$

$\langle 3\cos t, 3\sin t, 2 \rangle$

$\langle -3\sin t, 3\cos t, 0 \rangle$

↳ $\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{vmatrix}$

$= (3-0)i - (0-0)j + (-2-3)k$

$= 3i + 0j - 5k = \langle 3, 0, 5 \rangle$

curl $F \cdot dr = \langle 3, 0, 5 \rangle \cdot \langle -3\sin t, 3\cos t, 0 \rangle$

$\iint \text{curl} F \cdot dr = \boxed{45\pi}$

(13) $\Delta = (0,0)(3,0)(3,3)$

P Q R

$F = \langle y, z, x \rangle$

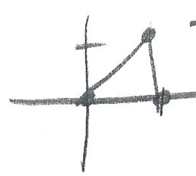
$(0,0,0)\ (3,0,0)\ (0,3,3)$

curl $F = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ y & z & x \end{vmatrix}$

curl $F = (0-1)i - (1-0)j + (0-1)k$

curl $F = \langle \overset{P}{-1}, \overset{Q}{-1}, \overset{R}{-1} \rangle$

curl $F = \langle 0, 0, 0 \rangle$



Type I region

$y = 3x \rightarrow$

$\int_0^3 \int_0^{3x}$

conservative vector field and is a closed curve so

$\boxed{\int \text{curl} F \cdot dr = 0}$