17.1 Homework

① $\oint_C xy\,dx + y\,dy$ unit circle: $r=1$

$x=\cos\theta,\ y=\sin\theta\ ;\ 0\le\theta\le 2\pi$

$\int_0^{2\pi} -\cos\theta\sin\theta\sin\theta\,d\theta + \sin\theta\cos\theta\,d\theta$

$\int_0^{2\pi}\left(-\cos\theta\sin^2\theta + \sin\theta\cos\theta\right)d\theta = 0$

Greene's Theorem: $\iint_D\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA$

$P(x,y) = xy\ ,\ Q(x,y) = y$

$P_y = x\ ,\ Q_x = 0$

$D = \{(x,y) \mid -1\le x\le 1,\ -\sqrt{1-x^2}\le y\le\sqrt{1-x^2}\}$

$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} -x\,dy\,dx = 0$

Greene's Theorem works!

③ $\oint_C y^2dx + x^2dy$

$P(x,y) = y^2,\ Q(x,y) = x^2$

$P_y = 2y\ ,\ Q_x = 2x$

$\iint_D (Q_x - P_y)\,dA = \iint_D (2x-2y)\,dA$

$\int_0^1\int_0^1 (2x-2y)\,dx\,dy = 0$

⑤ $\oint_C x^2y\,dx$

$P(x,y) = x^2y,\ Q(x,y) = 0$

$P_y = x^2\ ,\ Q_x = 0$

$\iint_D (Q_x - P_y)\,dA$

$D = \{(x,y) \mid -1\le x\le 1,\ -\sqrt{1-x}\le y\le\sqrt{1-x}\}$

$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} -x^2\,dy\,dx = -\frac{\pi}{4}$

⑦ $\oint_C F\cdot dr,\ F(x,y) = \langle x^2, x^2\rangle,\ C: y=x^2$ and $y=x$

$0\le x\le 1$

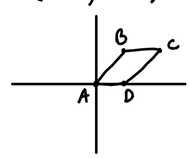
$\oint_C x^2dx + x^2dy$

$P(x,y) = x^2,\ Q(x,y) = x^2$

$P_y = 0\ ,\ Q_x = 2x$

$\int_0^1\int_{x^2}^{x} 2x\,dy\,dx = \frac{1}{6}$

⑨ $F = \langle e^{x+y}, e^{x-y}\rangle$



$A=(0,0),\ B=(2,2)$

$C=(4,2),\ D=(2,0)$

$\overline{AB}: y=x$ $\qquad \frac{2-0}{4-2} = 1$

$\overline{CD}: y=x-2$

$\oint_C e^{x+y}dx + e^{x-y}dy$

$P(x,y) = e^{x+y},\ Q(x,y) = e^{x-y}$

$P_y = e^{x+y},\ Q_x = e^{x-y}$

$\int_0^2\int_y^{y+2}\left(e^{x-y} - e^{x+y}\right)dx\,dy = -\frac{(e^2-1)(e^4-5)}{2}$

⑬ $I = \int_C (\sin x + y)\,dx + (3x+y)\,dy$

$\overline{DC}: y = -x+6$ $\qquad P(x,y) = \sin x + y\ ,\ Q(x,y) = 3x+y$

$\overline{AB}: y = x$ $\qquad P_y = 1,\ Q_x = 3$

$\{(x,y) \mid 0\le x\le 2,\ x\le y\le -x+6\}$

$\int_0^2\int_x^{-x+6} 2\,dy\,dx = 34$ idk.

17.1 Homework

① $F = \langle 2xy, x, y+z \rangle$, $z = 1-x^2-y^2$, $x^2+y^2 \leq 1$

Stoke's Theorem: $\int_C F \cdot dr = \iint_S \text{curl}(F)\, dS$

$\{(x,y) \mid -1 \leq x \leq 1, -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}\}$

$\text{curl } F = \langle 1, 0, 1-2x \rangle$

$\iint_S F\, ds = \iint_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right) dA$

$g(x,y) = 1-x^2-y^2$

$g_x = 2x$, $g_y = 2y$

$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (-2x + 1-2x)\, dy\, dx = \pi$

③ $F = \langle e^{y-z}, 0, 0 \rangle$ $0 \leq x \leq 1$, $0 \leq y \leq 1$, $z=1$

$\text{curl } F = \langle 0, -e^{y-z}, -e^{y-z} \rangle$

$z = 1 \Rightarrow \langle 0, -e^{y-1}, -e^{y-1} \rangle$

$\int_0^1\int_0^1 (-e^{y-1})\, dx\, dy = e^{-1} - 1$

⑤ $F = \langle e^{z^2} - y, e^{z^3} + x, \cos(xz) \rangle$, $x^2+y^2+z^2 = 1, z \geq 0$

$x = \cos t$, $y = \sin t$, $z = 0$: $r(t) = \langle \cos t, \sin t, 0 \rangle$

$F(r(t)) = \langle 1-\cos t, 1+\sin t, 1 \rangle$, $r'(t) = \langle -\sin t, \cos t, 0 \rangle$

$\int_0^{2\pi} (-\sin t + \sin t \cos t + \cos t + \sin t \cos t)\, dt$

$\int_0^{2\pi} (-\sin t + \cos t + 2\sin t \cos t)\, dt = 2\pi$ idk

$\text{curl } F = \langle -3e^{z^3} z^2, z(\sin(xz) + 2e^{z^2}), 2 \rangle$

(9) $\text{curl } F = \langle 0,0,0 \rangle$ $\iint_S \text{curl } F \cdot dS = 0$

(11) $F = \langle 3y, -2x, 3y \rangle$, $x^2 + y^2 = 9$, $z = 2$

$\text{curl } F = \langle 3, 0, -5 \rangle$

$\iint_S \text{curl } F \, dS = -45\pi$

(13) $F = \langle y, z, x \rangle$

$\{(x,y) \mid 0 \le x \le 3, 0 \le y \le 3\}$

$\iint_S \text{curl } F \, dS = 0$