17.1: 1 3 5 7 9 13
17.2: 1 3 5 9 11 13

Ch 17 HW

17.1

1) $\oint_C xy\,dx + y\,dy$ where C is the unit circle, CCW

$$\oint_{\partial D} F\cdot \underline{n}\,ds = \iint \mathrm{div}(F)\,dA$$

$dA = r\,dr\,d\theta$
$x = r\cos\theta$
$y = r\sin\theta$

$$= \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$= \iint_C (0 - x)\,dA = \int_0^{2\pi}\int_0^1 r\cos\theta \cdot r\,dr\,d\theta = \boxed{\int_0^{2\pi} \frac{\cos\theta}{3}\,d\theta}$$

3) $\oint y^2\,dx + x^2\,dy$ C: Unit square: $0 \le x \le 1$, $0 \le y \le 1$ CCW

$$= \boxed{\int_0^1\int_0^1 -(2x - 2y)\,dx\,dy}$$

ccw

5) $\oint x^2 y\,dx$ C: Unit circle cent@ origin CCW

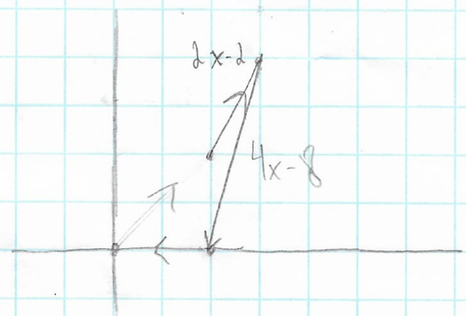
$$= \boxed{\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} -(0 - x^2)\,dx\,dy}$$

17.1 Cont

7) $\oint F \cdot d\underline{r} \quad F = \langle x^2, x^2 \rangle \quad C \quad \begin{matrix} x^2 \le y \le x \\ 0 \le x \le 1 \end{matrix}$

$$= \int_0^1 \int_{x^2}^{x} (2x - 0)\, dy\, dx = \boxed{\int_0^1 \int_{x^2}^{x} 2x\, dy\, dx}$$

9) $F = \langle e^{x+y}, e^{x-y} \rangle$



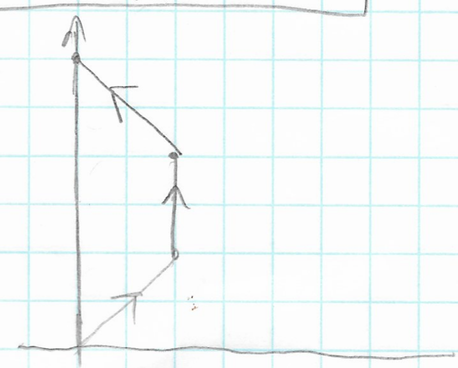
$$\text{curl}(F) = \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}$$

$$= e^{x-y} - e^{x+y}$$

$$e^x\left(\frac{1}{e^y} - e^y\right)$$

$\text{Area}(D) = 2 + (1 + \cancel{2}) - \cancel{2} = \boxed{3}$

Ans: $\boxed{\int_0^2 \int_0^{2x} e^x\left(\frac{1}{e^y} - e^y\right) dy\,dx + \int_2^3 \int_{4x-8}^{2x-2} e^x\left(\frac{1}{e^y} - e^y\right) dy\,dx}$



13) $I = \int_C (\sin x + y)\,dx + (3x + y)\,dy$

$$= \iint_C (3 - 1)\, dx\,dy + \int_0^6 2\, dy$$

$$2 \cdot 8 + 12 = \boxed{4}$$

Should be $\underline{34}$, could not find mistake

17.2

1) $F = \langle 2xy, x, y+z \rangle$ $\quad z = 1 - x^2 - y^2 \quad x^2 + y^2 \leq 1$

$z = 1 - r^2 \qquad r^2 \leq 1$

$r(t) = \langle \cos(t), \sin(t), 0 \rangle$

$r'(t) = \langle -\sin(t), \cos(t), 0 \rangle$

$$\iint_c F \cdot d\underline{s} \quad \int_0^{2\pi} 2\cos(t)\sin(t) \cdot -\sin(t) + \cos(t) \cdot \cos(t) + 0 \, dt = \boxed{\pi}$$

$$\text{curl}(F) = (z - 0)\hat{\imath} - (0 - 0)\hat{\jmath} + (1 - 2x)\hat{k}$$

$$= z\hat{\imath} + (1-2x)\hat{k} \qquad z = 0 \, ?\checkmark$$

$$\int \text{curl}(F) \cdot d\underline{S} = \int_{-1}^{1}\int_{\sqrt{1-x^2}}^{\sqrt{1-x^2}} (1-2x)\, dy\, dx = \boxed{\pi}$$

3) $F = \langle e^{y-z}, 0, 0 \rangle$

$$\text{curl}(F) = (0 - 0)\hat{\imath} - (0 + e^{y-z})\hat{\jmath} + (0 - e^{y-z})\hat{k}$$

$$= -e^{y-z}\hat{\jmath} - e^{y-z}\hat{k}$$

$C: \; z = 1 \quad 0 \leq x \leq 1, \; 0 \leq y \leq 1$

Ans $=$ $\int_0^1\int_0^1 0 \, dy\, dx = 0 \; ? \neq e^{-1} - 1$

17.2 Cont

3) $\iint_c F \cdot d\underline{s}$ $\quad r(t) = \langle t, t, 1 \rangle \quad 0 \le t \le 1$

$r'(t) = \langle 1, 1, 0 \rangle$

$= \int_0^1 -e^{t-1}(1)\,dt = \boxed{e^{-1} - 1}$

5) $F = \langle e^{z^2} - y, e^{z^3} + x, \cos(xz) \rangle$

$\text{curl}(F) = (0 - 3z^2 e^{z^3})\hat{i} - (-z\sin(xz) - 2ze^{z^2})\hat{j}$
$+ (1 - (-1))$

$\boxed{= -3z^2 e^{z^3}\hat{i} - (-z\sin(xz) - 2ze^{z^2})\hat{j} + 2\hat{k}}$

$x^2 + y^2 = 1 \quad \boxed{z = 0}$

$\text{curl}(F) = 0\hat{i} - 0\hat{j} + 2\hat{k}$

$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (\cos(0)) \cdot 2\hat{k} = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 2\,dy\,dx$

$= \boxed{2\pi}$

17.2 Cont

9) $F = \langle yz, xz, xy \rangle$ $\text{curl}(F) = 0$ (by inspection, $F = \nabla f$ where $f = xyz$)

$$\therefore \int_C \text{curl}(F) \cdot d\underline{S} = \boxed{0}$$

11) $F = \langle 3y, -2x, 3y \rangle$ C is $x^2 + y^2 = 9$ $z = 2$

$$\text{curl}(F) = (3-0)\hat{i} - (0-0)\hat{j} + (-2-3)\hat{k}$$

$$\iint_S F \cdot d\underline{S} = \qquad = 3\hat{i} - 5\hat{k}$$

$x = r\cos\theta$ $0 \le r \le 3$
$y = r\sin\theta$ $0 \le \theta \le 2\pi$
$z = 2$

$$\cancel{\iint_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right) dA}$$

$r(r,\theta) = \langle r\cos\theta, r\sin\theta, 2 \rangle$

$r_r = \langle \cos\theta, \sin\theta, 0 \rangle$ $r_\theta = \langle -r\sin\theta, r\cos\theta, 0 \rangle$

$|r_r \times r_\theta| = (0-0)\hat{i} - (0-0)\hat{j} + (r(\cos^2\theta + \sin^2\theta$

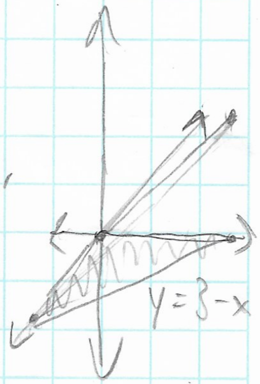
$$= \int_0^{2\pi}\int_0^3 (3\hat{i} - 5\hat{k}) \cdot d\underline{S}$$

$$d\underline{S} = \boxed{r\,dr\,d\theta\,\hat{k}}$$

$$= \int_0^{2\pi}\int_0^3 -5r\,dr\,d\theta = -45\pi$$

17.2 Cont

13) $F = \langle y, z, x \rangle$ C is triangle $\alpha(0,0,0)$, $\beta(3,0,0)$, $\gamma(0,3,3)$



curl(F) =

$(0-1)\hat{\imath} - (1-0)\hat{\jmath}$

$+ (0-1)$

$= \langle -1, -1, -1 \rangle$

$\alpha\beta = \vec{\beta}$

$\alpha\gamma = \vec{\gamma}$

$\vec{\beta} \times \vec{\gamma} = (0-0)\hat{\imath} - (9-0)\hat{\jmath}$

$+ (9-0)\hat{k}$

$= -9\hat{\jmath} + 9\hat{k}$

$= \langle 0, -9, 9 \rangle$

$\iint \text{curl}(F) \cdot dS$

$= \iint (-(-1)(0) - (-1)(1) - 1)\, dy\, dx$

$0(x-0) + (-9)(y-0) + 9(z-0) = 0$

$z - y = 0$

Explicit

$\boxed{z = y}$

$= \int_0^3 \int_0^{3-x} 0\, dy\, dx = 0$