homework 12

17.1

1.) closed int

$x = \cos\theta$

$y = \sin\theta$

$dx = -\sin\theta$

$\int_0^{\pi} (\cos\theta \sin\theta(-\sin\theta) + \sin\theta(\cos\theta))\, d\theta$ na

$$-\frac{\sin^3\theta}{3} - \frac{\cos^2\theta}{2}\Big|_0^{2\pi} = 0$$

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} -x \, dy\,dx = -xy\Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} = \int_{-1}^{1} -2x\sqrt{1-x^2}$$

$$\frac{2(1-x^2)^{3/2}}{3}\Big|_{-1}^{1} = 0$$

3.) $\oint_C y^2\,dx + x^2\,dy \quad 0 \le x \le 1 \; 0 \le y \le 1$

$P = y^2 \quad Q = x^2$

$Q_x = 2x \quad P_y = 2y$

$Q_x - P_y = 2x - 2y \rightarrow \iint_D 2x - 2y \, dA = 0$

5.) $\oint_C x^2 y\,dx$

$\cos\theta = x \quad \sin\theta = y$

$\cos^2\theta \sin\theta \, (-\sin\theta)\, d\theta$

$\theta_x = 0$

$\int_0^{2\pi} -\cos^2\theta \sin^2\theta \, d\theta \Rightarrow \frac{\sin(4\theta) - 4\theta}{32}\Big|_0^{2\pi} \rightarrow \frac{\pi}{4}$

7.) $\oint_C \langle x^2, x^2 \rangle$

$P_y = 0 \quad Q_x = 2x$

$\int_0^1 \int_x^{x^2} 2x \, dA = \int_0^1 \int_x^{x^2} 2x \, dy \, dx$

$\int_x^{x^2} 2xy \, dy \, dx \rightarrow 2xy \Big|_x^{x^2} \rightarrow 2x(x^2 - x)$

$\int_0^1 2(x-1)x^2 \rightarrow \frac{x^3(3x-4)}{6} \Big|_0^1 \rightarrow e \uparrow -\frac{1}{6}$

9.) $F(x,y) = \langle e^{x+y}, e^{x-y} \rangle$

clockwise!

$P_y = -e^{x-y}$

$Q_x = e^{x+y}$

$e^{x+y} - (-e^{x-y}) =$

$\int e^{x+y} + e^{x-y} \, dA$

(1,4) (3,4) (1,1) (3,1)

$x: 1 \rightarrow 3$

$y: 1 \rightarrow 4$

$\int_1^3 e^{x+y} + e^{x-y} \, dx =$

$\int_1^4 (e^2 - 1) e^{1-y} (e^2 + 1)$

13.) $\int (\sin x + y \, dx + 3x + y \, dy$

$P_y = \cos(y+x) \quad Q_x = 3$

$\int 3 - \cos(y+x)$

$x: 0 \rightarrow 2$

$y: x, 6-x \quad \int_x^{-x+6} 3 - \cos(y+x) \rightarrow 3y - \sin(y+x) \Big|_x^{-x+6} =$

$\int_0^2 \sin(2x) - 6x - \sin 6 + 18$

$I = 34$

17.2

1.) $F = \langle 2xy, x, y+z \rangle$

$z = 1 - x^2 - y^2$ for $x^2 + y^2 \leq 1$

curl: $\langle 1, 0, 1-2x \rangle$

$$\iint_0^{1-2x} \rightarrow \int_0^{2\pi} \int_0^{1 - r^2\cos^2(\theta)} r \, dr \, d\theta =$$

$$\int_0^{2\pi} \left(\frac{1}{2} + \frac{2}{3}\cos\theta\right) d\theta = \frac{1}{2}(2\pi) = \boxed{\pi}$$

3.) e^{y-z}

curl: $-e^{y-z}$

$$\iint -e^{y-z} \rightarrow e^{-1} - 1 = \boxed{\frac{1}{e} - 1}$$

5.) $\langle e^{z^2} - y, e^{z^3} + x, \cos(xz) \rangle$

curl: $\langle 0, z\sin(xz) + 2e^{z^2}, 3ex^3 x^2 + 2 \rangle$

$x^2 + y^2 + z^2 = 1 \quad z \geq 0$

$\langle -3z^2 e^{z^3}, 2z e^{z^2} + z\sin(xz), z \rangle$ $\int$ d

$z^2 = 4 : \sqrt{4} : 2$

$z = 2, \; x^2 + y^2 =$ $\int$ $= \boxed{2\pi}$

4.)

$\langle yz, xz, xy \rangle \rightarrow$ curl $= 0$

$0 \rightarrow 0 \rightarrow \boxed{0}$

11.) $\langle 3y, -2x, 3y \rangle \rightarrow$ curl $= \langle 3, 0, -5 \rangle$

$x^2+y^2=9$, $z=3$

$r=3$

$r(t) = \cos t\, i + \sin t\, j + 3k$

$0 \le t \le 2\pi$

$r'(t) = -\sin t\, i + \cos t\, j$

$\iint_S^{curl} F \cdot dS = \int_C F \cdot dr \quad \int_0^{2\pi}$ ~~$k\cos t \sin t + 3 \sin t \cos t \, dt$~~

$F(r(t)) = \langle 3(\sin t)i - 2(\cos t)j + 3(\sin t)k \rangle \cdot (-\sin t + \cos t)$

$\int_0^{2\pi}$ $= \boxed{-45\pi}$

13) $F = \langle y, z, x \rangle$ curl $= -1, -1, -1$

$x=0 \rightarrow$

$y=0 \rightarrow$ $\iint_S$ curl $F \cdot dS = -1(0) \rightarrow \boxed{0 = 10}$

$z=0 \rightarrow 3$

$(0,0,0)\ (3,0,0)\ (0,3,3)$

$\text{risk} = \frac{3}{6} \rightarrow$