

12/5/20. 17.1) Green's Theorem

#1, 3, 5, 7, 9, 13

1) $\oint xy dx + y dy$

$x = \cos \theta, y = \sin \theta$

$\int_0^{2\pi} -\cos \theta \sin \theta \sin \theta d\theta + \sin \theta d\theta$

$\iint_D \frac{dQ}{dx} - \frac{dP}{dy} dA$

$P = xy, Q = y$

$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x dy dx = 0$

3) $\oint_C y^2 dx + x^2 dy$

$0 \leq x \leq 1, 0 \leq y \leq 1$

$P = y^2, Q = x^2$

$P_y = 2y, Q_x = 2x$

$\iint_D (2x - 2y) dA$

$\int_0^1 \int_0^1 (2x - 2y) dx dy = 0$

5) $\oint_C x^2 y dx$

$P = x^2 y, Q = 0$

$P_y = x^2, Q_x = 0$

$\iint_D (Q_x - P_y) dA$

$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} -x^2 dy dx = \left(\frac{\pi}{4}\right)$

$$7) \oint_C \vec{F} \cdot d\vec{r} \quad \vec{F}(x,y) = \langle x^2, x^2 \rangle$$

$$C: y=x^2 \quad y=x$$

$$0 \leq x \leq 1$$

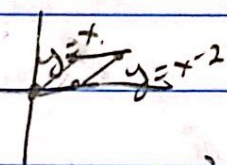
$$\oint_C x^2 dx + x^2 dy$$

$$P = x^2 \quad Q = x^2$$

$$P_y = 0 \quad Q_x = 2x$$

$$\int_0^1 \int_{x^2}^x 2x dy dx = \frac{1}{6}$$

$$9) \vec{F} = \langle e^{x+y}, e^{x-y} \rangle$$



$$P = e^{x+y} \quad Q = e^{x-y}$$

$$P_y = P \quad Q_x = Q$$

$$\int_0^2 \int_y^{y+2} (e^{x+y} - e^{x-y}) dx dy$$

$$\frac{(e^2 - 1)(e^4 - 5)}{2}$$

$$13) I = \int_C (\sin x + y) dx + (3x + y) dy$$

$$DC: y = -x + 6$$

$$P = \sin x + y$$

$$Q = 3x + y$$

$$AD: y = x$$

$$P_y = 1$$

$$Q_x = 3$$

$$\int_0^2 \int_x^{-x+6} 2 dy dx = 34$$

12/19/20 17.2 Stokes Theorem.

1, 3, 5, 9, 11, 13

1) $F = \langle 2xy, x+y+z \rangle$
 $z = 1 - x^2 - y^2$
 $x^2 + y^2 \leq 1$

$\int_C F \cdot dr = \iint_S \text{curl}(F) \cdot dS$

$\{ (x,y) \mid -1 \leq x \leq 1, -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2} \}$

$\text{curl}(F) = \langle 1, 0, 1-2x \rangle$

$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (-2x + 1 - 2x) dy dx = \pi$

3) $F = \langle e^{y-2}, 0, 0 \rangle$ $0 \leq x, y, z \leq 1$

$\text{curl}(F) = \langle 0, -e^{y-2}, -e^{y-2} \rangle$

When $z=1$, $\text{curl}(F) = \langle 0, -e^{y-1}, -e^{y-1} \rangle$

$\int_0^1 \int_0^1 (-e^{y-1}) dy dx = \frac{1}{e} - 1$

5) $F = \langle e^{z^2} - y, e^{z^2} + x \cos(xz) \rangle$
 $x^2 + y^2 + z^2 = 1$

$x = \cos(t)$ $y = \sin(t)$ $z = 0$

$r(t) = \langle \cos t, \sin t, 0 \rangle$

$F(r(t)) = \langle 1 - \cos t, 1 + \sin t, 1 \rangle$

$r'(t) = \langle -\sin t, \cos t, 0 \rangle$

$\int_0^{2\pi} (-\sin t + \sin t \cos t + \cos t + \cos t) dt$

C
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11) $F = \langle 3xy, -2x, 3y \rangle$

$\text{curl}(F) = \langle 3, 0, -5 \rangle$

$x^2 + y^2 = 9 \quad z = 2$

$r = 3$

$\iint_S \langle 3, 0, -5 \rangle \cdot dS = -45\pi$

13) $F = \langle y, z, x \rangle$

$\text{curl}(F) = \langle -1, -1, -1 \rangle$

$\iint_S \langle -1, -1, -1 \rangle \cdot dS = 0$