12/5/20. 17.1) Green's Theorem

#1, 3, 5, 7, 9, 13

1) $\oint xy\,dx + y\,dy$

$x = \cos\theta$, $y = \sin\theta$

$\int_0^{2\pi} -\cos\theta \sin\theta \sin\theta\,d\theta + \sin\theta\,d\theta$

$\iint_D \frac{dQ}{dx} - \frac{dP}{dy}\,dA$

$P = xy$, $Q = y$

$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x\,dy\,dx = 0$

3) $\oint_C y^2\,dx + x^2\,dy$

$0 \le x \le 1$ $\quad 0 \le y \le 1$

$P = y^2$ $\quad Q = x^2$

$P_y = 2y$ $\quad Q_x = 2x$

$\iint_D (2x - 2y)\,dA$

$\int_0^1 \int_0^1 (2x - 2y)\,dx\,dy = 0$

5) $\oint_C x^2 y\,dx$ $\quad \iint_D (Q_x - P_y)\,dA$

$P = x^2 y$ $\quad Q = 0$

$P_y = x^2$ $\quad Q_x = 0$ $\quad \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} -x^2\,dy\,dx = \frac{\pi}{4}$

7) $\oint_C F \cdot dr \quad F(x,y) = \langle x^2, x^2 \rangle$

$C: y = x^2 \qquad y = x$

$0 \le x \le 1$

$\oint_C x^2\,dx + x^2\,dy$

$P = x^2 \quad Q = x^2$

$P_y = 0 \quad Q_x = 2x$

$\int_0^1 \int_{x^2}^{x} 2x\,dy\,dx$

$= \frac{1}{6}$

9) $F = \langle e^{x+y}, e^{x-y} \rangle$

$y = x \quad y = x-2$

$P = e^{x+y} \quad Q = e^{x-y}$

$P_y = P \quad Q_x = Q$

$\int_0^2 \int_y^{y+2} (e^{x-y} - e^{x+y})\,dx\,dy$

$\frac{(e^2-1)(e^4-5)}{2}$

13) $I = \int_C (\sin x + y)\,dx + (3x+y)\,dy$

$\overline{DC} \quad y = -x+6 \qquad P = \sin x + y \qquad Q = 3x+y$

$\overline{AD}: y = x \qquad P_y = 1 \qquad Q_x = 3$

$\int_0^2 \int_x^{-x+6} 2\,dy\,dx = 34$

12/5/20 17.2 Stokes Theorem.

#1, 3, 5, 9, 11, 13

1) $F = \langle 2xy, x, y+z \rangle$

$z = 1 - x^2 - y^2$

$x^2 + y^2 \le 1$

$\int_C F \cdot dr = \iint \text{curl}(F)\, dS$

$\{(x,y) \mid -1 \le x \le 1, \ -\sqrt{1-x^2} \le y \le \sqrt{1-x^2}\}$

$\text{curl}(F) = \langle 1, 0, 1 - 2x \rangle$

$g = 1 - x^2 - y^2 \quad g_x = 2x, \ g_y = 2y.$

$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (-2x + 1 - 2x)\, dy\, dx = \pi.$

3) $F = \langle e^{y^{-2}}, 0, 0 \rangle \quad C: x, y, z = [0, 1]$

$\text{curl}(F) = \langle 0, -e^{y^{-2}}, -e^{y^{-2}} \rangle$

When $z = 1$, $\text{curl}(F) = \langle 0, -e^{y^{-1}}, -e^{y^{-1}} \rangle$.

$\int_0^1 \int_0^1 (-e^{y^{-1}})\, dy\, dx = \frac{1}{e} - 1$

5) $F = \langle e^{z^2} - y, e^{z^2} + x, \cos(xz) \rangle$

$x^2 + y^2 + z^2 = 1$

$x = \cos(t) \quad y = \sin(t) \quad z = 0.$

$r(t) = \langle \cos t, \sin t, 0 \rangle$

$F(r(t)) = \langle 1 - \cos t, 1 + \sin t, 1 \rangle$

$r'(t) = \langle -\sin t, \cos t, 0 \rangle$

$\int_0^{2\pi} (-\sin t + \sin t \cos t + \cos t + \sin t \cos t)\, dt$

11) $F = \langle 3y, -2x, 3y \rangle$.

$\text{curl}(F) = \langle 3, 0, -5 \rangle$

$x^2 + y^2 = 9 \quad z = 2$

$r = 3$

$\iint_S \langle 3, 0, -5 \rangle \, dS = -45\pi$.

13) $F = \langle y, z, x \rangle$.

$\text{curl}(F) = \langle -1, -1, -1 \rangle$

$\iint_S \langle -1, -1, -1 \rangle \, dS = 0$