17.1: 1-13 odd
17.2: 1-13 odd

17.1

1) $\oint xy\,dx + y\,dy$ $\quad x=\cos\theta \quad y=\sin\theta \quad 0\le t\le 2\pi$
$dx=-\sin\theta\,d\theta \quad dy=\cos\theta\,d\theta$

$\oint xy\,dx+y\,dy = \int_0^{2\pi} \cos\theta\sin^2\theta\,d\theta + \sin\theta\cos\theta\,d\theta = 0$ * using Maple

3) $\oint y^2dx + x^2dy \quad C: 0\le x\le 1, 0\le y\le 1$

$\vec{F} = \langle y^2, x^2\rangle \quad curl(\vec{F}) = 2x-2y$

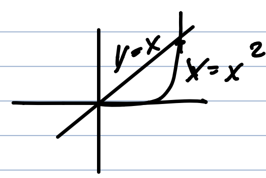
$\int_0^1\int_0^1 (2x-2y)\,dx\,dy = 0$

5) $\oint_C x^2y\,dx \Rightarrow \iint_D x^2\,dA$ $\quad x^2+y^2=1,\ r=1,\ 0\le\theta\le 2\pi$

$\int_0^{2\pi}\int_0^1 r^2\cos^2\theta\,dr\,d\theta = -\frac{\pi}{4}$

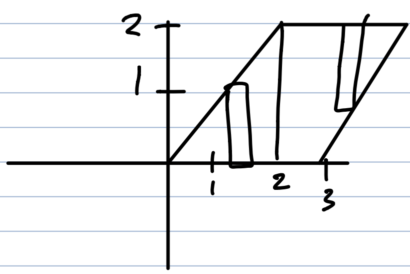
7) $\oint_C F\cdot d\vec{r} \Rightarrow \vec{F}(x,y) = \langle x^2, x^2\rangle \quad C: y=x^2, y=x, 0\le x\le 1$

$\int_0^1\int_{x^2}^{x} 2x\,dy\,dx$




$= \frac{1}{6}$ using maple

9) $\vec{F}(x,y) = \langle e^{x+y}, e^{x-y}\rangle \quad C: (0,0)\Rightarrow(2,2)\Rightarrow(4,2)\Rightarrow(2,0)\Rightarrow(0,0)$




$-\iint_D (e^{x-y}-e^{x+y})\,dA \Rightarrow -\int_0^2\int_0^x (e^{x-y}-e^{x+y}) =$

$\frac{5}{2} - \frac{1}{2}e^4 + \frac{1}{2}e^6 - \frac{5}{2}e^2$

11) $F = \langle 2xe^y, x + x^2e^y \rangle$ C: quarter circle path from A to B. Evaluate $I = \oint_C F \cdot dr$:

a) Find a function $f(x,y)$ where $\vec{F} = \vec{G} + \nabla f$, $G = \langle 0, x \rangle$

$\vec{F} - \vec{G} = \nabla f$

$\langle 2xe^y, x^2e^y \rangle = \nabla f$ $\qquad\qquad f(x,y) = x^2e^y$

$\int 2xe^y \, dx = x^2e^y + g(y)$

$\int x^2e^y \, dy = x^2e^y + g(x)$

b) Show that the the integrals of G along segements $\overline{OA}$ and $\overline{OB}$ are zero

$\int_{\overline{OA}} G \cdot dr = \int_0^1 \langle 0, x \rangle \cdot \langle 4, 0 \rangle dt = 0$

$r(t) = \langle 4t, 0 \rangle$

$x = 4t \quad y = 0$

$dx = dt(4) \quad dy = 0$

$\int_{\overline{OB}} G \cdot dr = \int_0^1 \langle 0, x \rangle \cdot \langle 0, 4 \rangle = 0$

$r(t) = \langle 0, 4t \rangle$

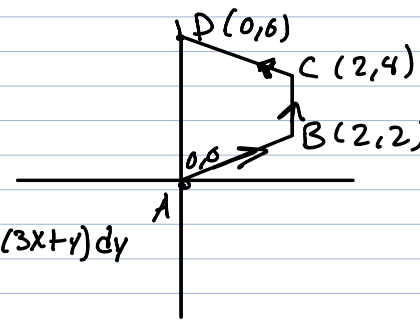
$dr = \langle 0, 4 \rangle dt$

$x = 0 \quad y = 4t$

$dx = 0 \quad dy = 4dt$

c) $I = 0 - 16 + 4\pi = 4\pi - 16$

13) $I = \int_C (\sin x + y)dx + (3x + y)dy$




$I = \int_{SD} (\sin x + y)dx + (3x + y)dy - \int_{DA} (\sin x + y)dx + (3x + y)dy$

$= 34$ # using maple

17.2

1) $F=\langle yz, 0, x\rangle$, $z=1-x^2-y^2$ for $x^2+y^2\le 1$

$C(t)=\langle \cos t, \sin t, 0\rangle,\ 0\le t\le 2\pi$
$C'(t)=\langle -\sin t, \cos t, 0\rangle,$

$F(C(t))\cdot c'(t) = -2\sin^2 t\cos t+\cos^2 t$

$\int_0^{2\pi} -2\sin^2 t\cos t+\cos^2 t = \pi$

$T_\theta=\langle -t\sin\theta, t\cos\theta, 0\rangle$
$T_t=\langle \cos\theta, \sin\theta, -2t\rangle$

$T_\theta\times T_t=\langle -2t^2\cos\theta, -2t^2\sin\theta, t\rangle = n$

$curl(F)=\langle 1, 0, 1-2x\rangle$

$curl(F)\cdot\vec{n}=t$

$\int_0^{2\pi}\int_0^1 t\,dt\,d\theta=\pi$

3) $F=\langle e^{y-z}, 0, 0\rangle$, $[\langle 1,0,1\rangle, \langle 1,1,1\rangle \langle 0,1,1\rangle, \langle 0,0,1\rangle]$

$z=2$

AB: $x=1$ y varies from 0 to 1
BC: $y=1$ x varies from 1 to 0
CD: $x=0$ y varies from 1 to 0
DA: $y=0$ x varies from 0 to 1

$\iint_C curl(\vec{F})\cdot d\vec{S} = \iint_S curl(F)\cdot\vec{n}\,d\vec{S}$

$curl(F)=\langle 0, e^{y-z}, -e^{y-z}\rangle \quad \vec{n}=\vec{k}$

$curl(F)\cdot n\,dS = e^{y-2}$

$\int_0^1\int_0^1 (-e^{y-1})\,dx\,dy = e^{-1}-1$

$\boxed{e^{-1}-1}$ →

Stokes

$\iint_S curl(F)\cdot dS = \oint_{\partial S} F\cdot ds$

$=\oint_{AB} F\cdot dS+\oint_{BC} F\cdot dS+\oint_{CD} F\cdot ds+\oint_{DA} F\cdot ds$

$\oint_{AB} F\cdot ds = \int_{AB} e^{y-2}\,dx = 0$

$\oint_{BC} F\cdot ds = \int_{BC} e^{y-2}\,dx = \int_1^0 dx = -1$

$\oint_{CD} F\cdot ds = \int_{CD} e^{y-2}\,dx = 0$

$\oint_{DA} F\cdot dS = \int_0^1 e^{-1}\,dx = e^{-1}$

5) $F = \langle e^{z^2} - y,\ e^{z^3} + x,\ \cos(xz) \rangle$, upper hemisphere $x^2+y^2+z^2=1$ $z \geq 0$ with outward pointing normal

$\text{curl}(F) = \langle -3z^2 e^{z^3},\ z\sin(xz) + 2ze^{z^2},\ 2 \rangle$

$\iint_S \text{curl}(\vec{F}) \cdot d\vec{S} = \oint_{\partial S} F \cdot ds$

$C(t) = \langle \cos t, \sin t, 0 \rangle$ $0 \leq t \leq 2\pi$

$C'(t) = \langle -\sin t, \cos t, 0 \rangle$

$F(C(t)) \langle 1 - \sin t, \cos t + 1, 1 \rangle$

$\int_0^{2\pi} (1 - \sin t + \cos t)\, dt = 2\pi$

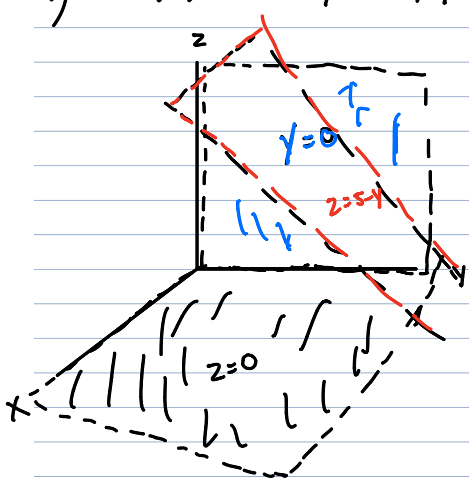
7) $F(x,y,z) = \langle xy^2, yz^2, zx^2 \rangle$, S: boundary of the cylinder given by $x^2+y^2 \leq 4$, $0 \leq z \leq 3$

$0 \leq r \leq 2$ $0 \leq z \leq 3$
$0 \leq \theta \leq 2\pi$

$\iiint \text{div}(F) \cdot dV \Rightarrow \iiint y^2 + z^2 + x^2 \cdot dV$

$\int_0^3 \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (y^2 + z^2 + x^2)\, dy\, dx\, dz = 60\pi$

9) $F(x,y,z) = \langle x + z^2,\ xz + y^2,\ xz - y \rangle$ ; $z = 1 - x^2$, $z = 0$, $y = 0$, $z + y = 5$




$D = (\{x,y,z\} \mid 0 \leq z \leq 1 - x^2,\ 0 \leq y \leq x^2 - 4,\ -2 \leq x \leq 2$

$5 - y = 1 - x^2$

$x^2 - 4 = y$

$\int_{-2}^{2} \int_0^{1-x^2} \int_0^{y^2-4} \text{div}(F)\, dy\, dz\, dx = \frac{8866}{105}$

11) $F(x,y,z) = \langle x^3, 0, z^3 \rangle$ ; $x^2+y^2+z^2 \leq 4$ ; $x, y, z \geq 0$

$\iiint \text{div}(F)\, dV \Rightarrow \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^2 \text{div}(F(\text{spherical coordinates})) \cdot \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta = \frac{32\pi}{5}$

$D = (\{\theta, \phi, z\} = 0 \leq \theta, 0 \leq \frac{\pi}{2}$ $0 \leq \omega \leq 2$ )

13) $F(x,y,z) = \langle x, y^2, z+y \rangle$ . $x^2+y^2=4$ $z = x$ and $z=8$

$\iiint \text{div}(F)\,dV$ $D = (\{x,y,z\} \mid x \le z \le 8,\ 0 \le \theta \le 2\pi\ 0 \le r \le 2$

¿?? confused