1. $\oint_C (xy\,dx + y\,dy)$

$x = r\cos t,\ y = r\sin t \quad r=1,\ t \in [0, 2\pi]$

$$\int_0^{2\pi} (r\cos t \cdot r\sin t \cdot r(-\sin t)\,dt + r\sin t \cdot r\cos t\,dt)$$

$$= \int_0^{2\pi} -\sin^2 t\, d(\sin t) + \int_0^{2\pi} \sin t\, d(\sin t)$$

$$= 0$$

For Green Theorem

$$\iint_D (0-x)\,dx\,dy = -\int_0^{2\pi}\int_0^1 r\cos\theta\, r\,dr\,d\theta$$

$$= -\frac{1}{3}(\sin\theta)\Big|_0^{2\pi} = 0$$

3. $\oint_C y^2 dx + x^2 dy$, $0 \le x \le 1$, $0 \le y \le 1$

$\frac{d}{dy} = 2y$, $\frac{d}{dx} = 2x$

$$= \int_0^1\int_0^1 (2x - 2y)\,dy\,dx$$

$$= 2\int_0^1\int_0^1 (x-y)\,dy\,dx$$

$$= 2\left(\frac{1}{2} - \frac{1}{2}\right) = 0$$

5. $\oint_C x^2 y\,dx$

$P = x^2 y$, $Q = 0$

$\frac{dQ}{dx} - \frac{dP}{dy} = -x^2$

$$I = \int_C x^2 y\,dx = \iint_D -x^2\,dA = \int_0^{2\pi}\int_0^1 -r^3\cos^2\theta\,dr\,d\theta$$

$$= -\frac{\pi}{4}$$

7. $\oint_C F\cdot dr \quad F(x,y)=\langle x^2, x^2\rangle$

$F(x,y)=\langle x^2,x^2\rangle$

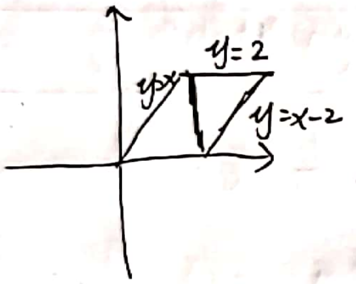
$\frac{dQ}{dx}=2x, \quad \frac{dP}{dy}=0, \quad$ and $x^2\leq y\leq x^2, \ 0\leq x\leq 1$

$\oint_C F\cdot dr = \int_0^1\int_{x^2}^{x^2}(2x)\,dy\,dx$

$= \frac{1}{6}$

9. $F(x,y)=\langle e^{x+y}, e^{x-y}\rangle$

$\frac{dQ}{dx}=e^{x-y}, \quad \frac{dP}{dy}=e^{x+y}$



$\int_0^2\int_0^x (e^{x-y}-e^{x+y})\,dy\,dx + \int_2^4\int_{x-2}^{2}(e^{x-y}-e^{x+y})\,dy\,dx$

$= \frac{(e^2-1)(e^4-5)}{2}$

13. $I=\int_C (\sin x+y)dx+(3x+y)dy$

$P=\sin x+y, \quad Q=3x+y$

$\frac{dQ}{dx}=3, \quad \frac{dP}{dy}=1$

$\iint_D (3-1)\,dA=-2$

$A=\frac{1}{2}(2+6)\cdot 2 = 8$

$\int_C (\sin x+y)dx+(3x+y)dy)=-16$

$\int_C (\sin(0)+t)\cdot 0\,dt + (3(0)+t)dt = \int_6^0 t\,dt = -18$

$-16+(-18)=-34$

1. $F = \langle 2xy, x, y+z \rangle$, $z = 1 - x^2 - y^2$ for $x^2+y^2 \le 1$

$\nabla f = \langle 2x, 2y, 1 \rangle$ $\quad f(x,y,z) = z + x^2 + y^2 - 1$

$\|\nabla f\| = \sqrt{4x^2+4y^2+1}$

$\text{curl}\, F = \langle 1, 0, 1-2x \rangle$

$dS = \sqrt{4x^2+4y^2+1}$

$$\iint_S \text{curl}\, F \cdot r\, ds = \iint_D \frac{\langle 1,0,1-2x \rangle \langle 2x, 2y, 1 \rangle}{\sqrt{4x^2+4y^2+1}} \cdot dS\, dA$$

$$= \iint_D (2x + 1 - 2x)\, dA$$

$$= \iint_D 1 \cdot dA$$

$D = \{ (r,\theta) \mid 0 \le \theta \le 2\pi, 0 \le r \le 1 \}$

$$= \int_0^{2\pi} d\theta \int_0^1 r\, dr$$

$$= \pi$$

3. $F = \langle e^{y-z}, 0, 0 \rangle$

$\text{curl}\,(F) = (0, -e^{y-z}, -e^{y-z})$

$$\iint (\text{curl}\, F) \cdot ds$$

$$= \int_0^1 \int_0^1 -e^{y-z}\, dy\, dx$$

$$= e^{-1} - 1$$

5. $F = \langle e^{z^2} - y, e^{z^3} + x, \cos(xz) \rangle$, $x^2 + y^2 + z^2 = 1$, $z \geq 0$

curl $F = (-3e^{z^3} z^2, z(\sin(xz) + 2e^{z^2}), 2)$

$\iint_S \text{curl } F \cdot dS = \int_C \cancel{-3e^{z^3} z^2 dx +} (e^{z^2} - y) dx + (e^{z^3} + x) dy + \cos(xz) dz$

$x = \cos t$, $y = \sin t$, $z = 0$, $0 \leq t \leq 2\pi$

$\iint_S \text{curl } F \cdot dS = \int_0^{2\pi} (1 - \sin t)(-\sin t \, dt) + (1 + \cos t)(\cos t \, dt)$

$= (\cos t + \sin t + t) \Big|_0^{2\pi}$

$= 2\pi$

9. $F = \langle yz, xz, xy \rangle$

curl$(F) = (0, 0, 0) = 0$

$\oint F dr = \iint_S (\text{curl} F \cdot N) dS$

$= \iint_{x^2+y^2=1} 0 \, dS$

$= 0$

11. $F = \langle 3y, -2x, 3y \rangle$

curl$(F) = \langle 3, 0, -5 \rangle$

$F \cdot n = \langle 3, 0, -5 \rangle \cdot \langle 0, 0, 1 \rangle = -5$

$\oint_C F \cdot dr = \iint_S \text{curl } F \cdot n \, dS = \iint_S (-5) dS$

$= -5(\pi \cdot 3^2) = -45\pi$

13. $F = \langle y, z, x \rangle$

$\text{curl}(F) = \langle -1, -1, -1 \rangle$

$PQ = \langle 3, 0, 0 \rangle \quad PR = \langle 0, 3, 3 \rangle$

$PQ \times PR = \langle 0, -9, 9 \rangle$

$S = \{ (x, y, z) \mid y - z = 0 \}$

$r(u, v) = ui + vj + vk$

$r_u(u, v) = i$

$r_v(u, v) = j + k$

$r_u \times r_v = \langle 0, -1, -1 \rangle$

$$\oint_C F \cdot dr = \iint_S (\text{curl}(F) \cdot n \, dS$$

$$= \iint 0 \, dx dy$$

$$= 0$$

扫描全能王 创建