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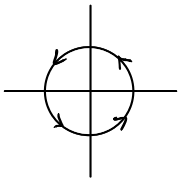
17.1 : 1, 3, 5, 7, 9, 13

#1 $\oint_C xy\,dx + y\,dy$



$0 < \theta < 2\pi$

$x = \cos\theta \quad dx = -\sin\theta\, d\theta$

$y = \sin\theta \quad dy = \cos\theta\, d\theta$

$xy\,dx + y\,dy \longrightarrow \cos\theta\sin\theta(-\sin\theta\, d\theta) + \sin\theta\cos\theta\, d\theta = \int \sin\theta\cos\theta - \cos\theta\sin^2\theta\, d\theta$

$\int_0^{2\pi} \sin\theta\cos\theta - \cos\theta\sin^2\theta\, d\theta =$

$\rightarrow \int_0^{2\pi} \frac{\sin(2\theta)}{2} \quad u = 2\theta \quad du = 2d\theta \rightarrow \frac{1}{4}\int_0^{4\pi} \sin u\, du : \frac{1}{4}(-\cos u)\Big|_0^{4\pi} : -\frac{1}{4} + \frac{1}{4} = 0$

$\sin^2 + \cos^2 = 1$

$\sin^2 = 1 - \cos^2$

$\rightarrow \int_0^{2\pi} \cos\theta(1-\cos^2\theta)\, d\theta \quad u = 1-\cos^2\theta \quad u' = -2\cos\theta \cdot \sin\theta \quad v = \sin\theta \quad v' = \cos\theta \rightarrow (1-\cos^2\theta)\cos\theta + 2\int \cos\theta\sin\theta\sin\theta\, d\theta$

$u = \sin\theta \quad du = \cos\theta\, d\theta$

$= \cos\theta - \cos^3\theta + \int u^2\, du = \cos\theta - \cos^3\theta + \frac{\sin^3\theta}{3}\Big|_0^{2\pi} = (1 - 1 + 0) - (1 - 1 + 0) = 0$

$0 - 0 = \boxed{0}$

Now with greens theorem:

$$\oint F_1 dx + F_2 dy = \iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dA$$

$\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 0 - x = -x$

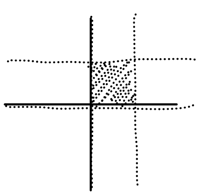
~~$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x\, dx\, dy$~~

since the area is a circle at center 0, the area is 0 because the areas of the top and bottom hemisphere cancel out.

$0 = 0$ ✓

#3 $\oint_C y^2 dx + x^2 dy \qquad 0 \le x \le 1,\ 0 \le y \le 1$

$\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 2x - 2y$



$x^2 - 2yx\Big|_0^1 : 1 - 2y - 0$

$\int_0^1\int_0^1 2x - 2y\, dx\, dy = \int_0^1 \left[\int_0^1 2x - 2y\, dx\right] dy = \int_0^1 1 - 2y\, dy = y - y^2\Big|_0^1$

$= 0$

#5 $\oint_C x^2 y\,dx$ $\qquad \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 0 - x^2$ $\qquad 0 < r < 1$, $0 < \theta < 2\pi$

$$\rightarrow \int_0^{2\pi}\int_0^1 r^2\cos^2\theta\, r\,dr\,d\theta$$

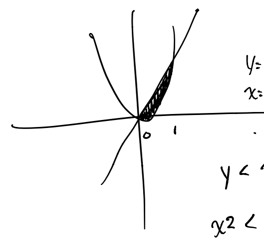
$$\int_0^1 r^2\cos^2\theta\, r\,dr = \frac{r^4}{4}\cos^2\theta \Big|_0^1 = \frac{\cos^2\theta}{4}$$

$$\frac{1}{4}\int_0^{2\pi}\cos^2\theta\,d\theta = \frac{1}{8}\int 1+\cos(2\theta)\,d\theta = \frac{\theta}{8} + \frac{\sin(2\theta)}{16}\Big|_0^{2\pi} = \left[\frac{\pi}{4}+0\right]-\left[0+0\right]$$

$$= \boxed{\frac{\pi}{4}}$$

#7 $\oint F\cdot dr$ where $F(x,y) = \langle x^2, x^2\rangle$ and C consists of the arcs $y=x^2$ and $y=x$ for $0\le x\le 1$

$$\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 2x - 0 = 2x$$



$y < x < \sqrt{y}$

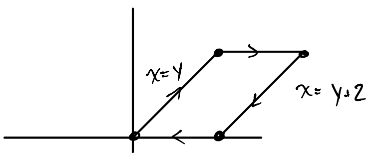
$x^2 < y < x$

$$\rightarrow \int_0^1\int_{x^2}^{x} 2x\,dy\,dx$$

$$\int_{x^2}^{x} 2x\,dy = 2xy\Big|_{x^2}^{x} = 2x^2 - 2x^3$$

$$\int_0^1 2x^2 - 2x^3\,dx = \frac{2x^3}{3} - \frac{2x^4}{4}\Big|_0^1 = \frac{2}{3} - \frac{2}{4} = \boxed{\frac{1}{6}}$$

#9 $F(x,y) = \langle e^{x+y}, e^{x-y}\rangle$



$$F_{2x} - F_{1y} = e^{x-y} - e^{x+y} = \frac{e^x}{e^y} - e^x e^y = e^x\left(e^{-y} - e^{y}\right)$$

$$\int_0^2\int_y^{y-2} e^x\left(e^{-y}-e^{y}\right)dx\,dy =$$

$$\rightarrow \int_y^{y-2} e^x\left(e^{-y}-e^{y}\right)dx = e^x\left(e^{-y}-e^{y}\right)\Big|_y^{y-2}$$

$$\int_0^2 e^{y-2}\left(e^{-y}-e^{y}\right) - e^{y}\left(e^{-y}-e^{y}\right)dy = \int_0^2 e^{-2} - e^{2y-2} - e^0 + e^{2y}\,dy$$

$$= ye^{-2} - \frac{e^{2y-2}}{2} - y + \frac{e^{2y}}{2}\Big|_0^2 = \left[e^{-2} - \frac{e^2}{2} - 2 + \frac{e^4}{2}\right] - \left[0 - \frac{e^{-2}}{2} + \frac{1}{2}\right] =$$

$$= e^{-2} - \frac{e^2+e^4+e^{-2}}{2} - \frac{5}{2} = \boxed{e^{-2} - \frac{e^4-5}{2}}$$

13 $I = \int_C (\sin x + y)\,dx + (3x+y)\,dy$ for nonclosed ABCD

$F_{2x} - F_{1y} = 3 - 1 = 2$

Area $= \frac{(2+6)2}{2} = 8$ $\iint 2\,dA = 2 * \text{Area} = 16$

DA parametrization: $\langle 0, t \rangle$ $r'(t) = \langle 0, 1 \rangle$

$\int_0^6 \langle 0,t \rangle \cdot \langle 0,1 \rangle = 0 + t\ dt$

$\int_0^6 t\,dt = \frac{t^2}{2}\Big|_0^6 = 18 - 0 = 18$

$16 - 18 = \boxed{-2}$

17.2 1, 3, 5, 9, 11, 13

#1 $F = \langle 2xy, x, y+z \rangle$ surface $z = 1 - x^2 - y^2$ for $x^2 + y^2 \le 1$

$r(t) = \langle \cos t, \sin t, 0 \rangle \quad 0 < t < 2\pi$

$F(r(t)) = \langle 2\cos t \sin t, \cos t, \sin t \rangle$

$r'(t) = \langle -\sin t, \cos t, 0 \rangle$

$\langle 2\cos t\sin t, \cos t, \sin t \rangle \cdot \langle -\sin t, \cos t, 0 \rangle = \langle -2\cos t \sin^2 t, \cos^2 t, 0 \rangle$

$$\int_0^{2\pi} -2\cos t \sin^2 t + \cos^2 t \, dt$$

$$\frac{-2}{2}\int_0^{2\pi} \cos t \cdot (1 - \cos(2t)) \, dt = -\int \cos t (1 - \cos 2t) \, dt \qquad u = 1 - \cos 2t \quad v = \sin t \qquad u' = -2\sin 2t \quad v' = \cos t$$

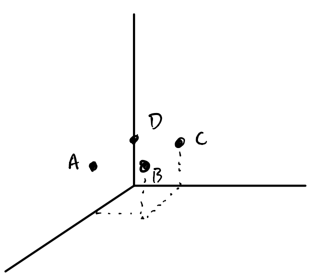
$$\to \cos t \sin t - \int -2\sin 2t \cdot \sin t \, dt = \cos t \sin t - \int -2(2\cos t \sin t)\sin t =$$

$$= \cos t \sin t + 2\int 2\cos t \sin t \sin t \, dt = \cos t \sin t + 4\int \cos t \sin^2 t \, dt \qquad u = \sin t \quad du = \cos t \, dt$$

$$\to \int_0^0 u^2 \, du = \frac{u^3}{3}\Big|_0^0 = 0$$

$$\int_0^{2\pi} \cos^2 t \, dt = \frac{1}{2}\int 1 + \cos(2t) \, dt = t + \frac{\sin(2t)}{2}\Big|_0^{2\pi} = \left[2\pi + 0\right] - \left[0 + 0\right] = \frac{1}{2}(2\pi) = \boxed{\pi}$$

#3 $F = \langle e^{y-z}, 0, 0 \rangle$ square $(1,0,1)$, $(1,1,1)$, $(0,1,1)$ and $(0,0,1)$
A B C D



$r_1(t) = \overline{AB} = (1-t)(1,0,1) + t(1,1,1) = (1-t, 0, 1-t) + (t,t,t) = \langle 1, t, 1 \rangle$
$r_2(t) = BC = (1-t)(1,1,1) + t(0,1,1) = (1-t, 1-t, 1-t) + (0,t,t) = \langle 1-t, 1, 1 \rangle$
$r_3(t) = CD = (1-t)(0,1,1) + t(0,0,1) = (0, 1-t, 1-t) + (0,0,t) = \langle 0, 1-t, 1 \rangle$
$r_4(t) = DA = (1-t)(0,0,1) + t(1,0,1) = (0,0,1-t) + (t,0,t) = \langle t, 0, 1 \rangle$

$F(r_1(t)) = \langle e^{t-1}, 0, 0 \rangle$
$F(r_2(t)) = \langle e^0, 0, 0 \rangle = \langle 1, 0, 0 \rangle$
$F(r_3(t)) = \langle e^{1-t-1}, 0, 0 \rangle = \langle e^{-t}, 0, 0 \rangle$
$F(r_4(t)) = \langle e^{0-1}, 0, 0 \rangle = \langle e^{-1}, 0, 0 \rangle$

$F(r_1(t)) \cdot r_1'(t) = \langle e^{t-1}, 0, 0 \rangle \cdot \langle 0, 1, 0 \rangle = 0$
$F(r_2(t)) \cdot r_2'(t) = \langle 1, 0, 0 \rangle \cdot \langle -1, 0, 0 \rangle = -1$
$F(r_3(t)) \cdot r_3'(t) = \langle e^{-t}, 0, 0 \rangle \cdot \langle 0, -1, 0 \rangle = 0$
$F(r_4(t)) \cdot r_4'(t) = \langle e^{-1}, 0, 0 \rangle \cdot \langle 1, 0, 0 \rangle = e^{-1}$

$$\int_0^1 0\,dt + \int_0^1 -1\,dt \int_0^1 0 + \int_0^1 e^{-1}dt = -\int_0^1 dt + \int_0^1 e^{-1}dt$$

$\overset{\uparrow}{0}$ $\qquad\qquad\qquad$ $\overset{|}{0}$

$$= -t\Big|_0^1 + te^{-1}\Big|_0^1 = -1 + e^{-1}$$

$$= \boxed{e^{-1} - 1}$$

#5 $F = \langle e^{z^2} - y,\ e^{z^3} + x,\ \cos(xz)\rangle$ upper hemisphere $x^2 + y^2 + z^2 = 1$ $\quad z \geq 0$

$r(t) = \langle \cos t, \sin t, 0\rangle \qquad 0 < t < 2\pi$

$r'(t) = \langle -\sin t, \cos t, 0\rangle$

$F(r(t)) = \langle e^0 - \sin t,\ e^0 + \cos t,\ 0\rangle = \langle 1 - \sin t,\ 1 + \cos t,\ 0\rangle$

$\langle 1 - \sin t, 1 + \cos t, 0\rangle \cdot \langle \cos t, \sin t, 0\rangle = \cos t - \sin t\cos t + \sin t + \sin t\cos t$

$$\int_0^{2\pi} \cos t + \sin t\,dt = \sin t - \cos t\Big|_0^{2\pi} = \overset{-1-(-1)}{(0-1)-(0-1)} = 0$$

ignore this sorry I didn't read the question.

$$\text{Curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{z^2} - y & e^{z^3} + x & \cos xz \end{vmatrix}$$

$\rightarrow (0 - e^{z^3} \cdot 3z^2)i - (-\sin xz \cdot z - e^{z^2} \cdot 2z)j + (1+1)k$

$= -3z^2e^{z^3} + z\sin(xz) + 2ze^{z^2} + 2$

#9 $\quad F = \langle yz, xz, xy\rangle$

$$\text{Curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} = (x - x)i - (y - y)j + (z - z)k = 0$$

#11 $F = \langle 3y, -2x, 3y \rangle$ C is circle $x^2+y^2=9$, $z=2$

$r(t) = \langle 3\cos t, 3\sin t, 2 \rangle$

$r'(t) = \langle -3\sin t, 3\cos t, 0 \rangle$

$F(r(t)) = \langle 9\sin t, -6\cos t, 6 \rangle$

$F(r(t)) \cdot r'(t) = \langle 9\sin t, -6\cos t, 6 \rangle \cdot \langle -3\sin t, 3\cos t, 0 \rangle = -27\sin^2 t - 18\cos^2 t$

$$\int_0^{2\pi} -27\sin^2 t - 18\cos^2 t \, dt = \int_0^{2\pi} -27(1-\cos^2 t) - 18\cos^2 t \, dt$$

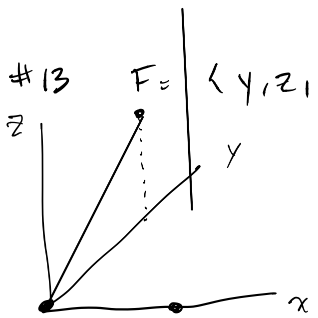
$$= \int_0^{2\pi} -27 + 27\cos^2 t - 18\cos^2 t \, dt = \int_0^{2\pi} -27 + 9\cos^2 t \, dt$$

$$= \int_0^{2\pi} -27 \, dt + \frac{9}{2}\int 1 + \cos(2t) \, dt = -27t \Big|_0^{2\pi} + \frac{9}{2}\left(t + \frac{\sin(2t)}{2}\right)\Big|_0^{2\pi}$$

$$= -54\pi + \frac{9}{2}\left([2\pi + 0] - [0+0]\right)$$

$$= -54\pi + 9\pi = \boxed{-45\pi}$$

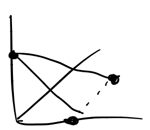
#13 $F = \langle y, z, x \rangle$ $A(0,0,0)$, $B(3,0,0)$, $C(0,3,3)$



$AB = \langle 3,0,0 \rangle$

$AC = \langle 0,3,3 \rangle$

$$\begin{matrix} i & j & k \\ 3 & 0 & 0 \\ 0 & 3 & 3 \end{matrix} \quad (0)i - (9-0)j + (9-0)k = \langle 0,-9,9 \rangle$$



$0(x-0) - 9(y-0) + 9(z-0) = 0$

$-9y + 9z = 0$ $9z = 9y$ $z = y$

$g(x,y) = y$

$$\text{Curl}F: \begin{matrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z & x \end{matrix} = (0-1)i - (1-0)j + (0-1)k = -i - j - k$$

$$\int +1(0) + 1(1) + (-1) \, dA = \boxed{0}$$

$0 + 1 - 1$