17.1

1. $\oint_C xy\,dx + y\,dy$, C is unit circle, counterclockwise

$\iint \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA = \iint 0 - x\, dA$

$\int_0^{2\pi}\int_0^1 -r\cos\theta\, r\,dr\,d\theta = \frac{-r^3}{3}\cos\theta \Big|_0^1 = \int_0^{2\pi} \frac{-1}{3}\cos\theta\, d\theta$

$\frac{-1}{3}\int_0^{2\pi} \cos\theta\, d\theta = \frac{1}{3}\sin\theta \Big|_0^{2\pi} = \frac{1}{3}(\sin 2\pi - \sin 0) = 0$

$\int_0^{2\pi} (\cos\theta \sin\theta(-\sin\theta) + \sin\theta\cos\theta)\, d\theta = 0$

3. $\oint y^2 dx + x^2 dy$ C is $0 \le x \le 1$ $0 \le y \le 1$

$\int_0^1\int_0^1 2x - 2y\, dy\, dx = 2xy - y^2 \Big|_0^1 = 2x - 1$

$\int_0^1 2x - 1\, dx = x^2 - x \Big|_0^1 = 1 - 1 = 0$

5. $\oint x^2 y\, dx$, C is unit circle

$\iint 0 - x^2\, dA = -1\int_0^{2\pi}\int_0^1 r^3\cos^2\theta\, dr\, d\theta$

$\frac{-1}{4}\int_0^{2\pi} \cos^2\theta = \frac{-1}{4}\left(\frac{1}{2}\theta + \frac{\sin 2\theta}{4}\Big|_0^{2\pi}\right)$

$= \frac{-1}{4}\left(\frac{2\pi}{2} + \frac{\sin 4\pi}{4}\right) - 0$

$= \frac{-\pi}{4}$

7. $\oint F \cdot dr$, $F(x,y) = \langle x^2, x^2 \rangle$ C is $y = x^2$ and $y = x$ $0 \le x \le 1$

$x \le y \le x^2$

$0 \le x \le 1$

$\int_0^1\int_x^{x^2} 2x - 0\, dy\, dx$

$\int_0^1 2xy \Big|_x^{x^2} dx = \int_0^1 2x^3 - 2x^2\, dx$

$= \frac{2x^4}{4} - \frac{2x^3}{3} \Big|_0^1 = \left(\frac{6}{12} - \frac{8}{12}\right) - 0 = \frac{-1}{6} \cdot -1 = \frac{1}{6}$

9. $F(x,y) = \langle e^{x+y}, e^{x-y} \rangle$, clockwise, (0,0) (2,2) (4,2) (2,0)

$$\int F \cdot dr = \int_0^4 \int_0^2 e^{x-y} - e^{x+y} \, dy\,dx$$

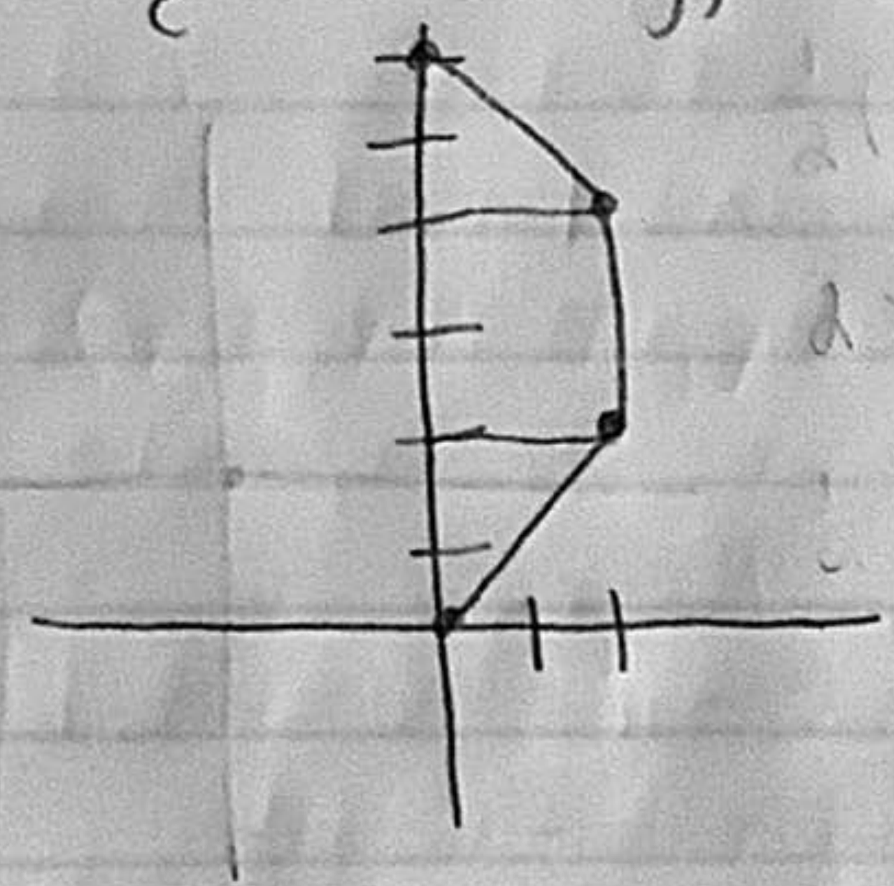
$$\int_0^4 -e^{x-y} - e^{x+y} \Big|_0^2 \, dx = \int_0^4 -e^{x-2} - e^{x+2} + e^x + e^x \, dx$$

$$-e^{x-2} - e^{x+2} + e^x + e^x \Big|_0^4 = (-e^2 - e^6 + e^4 + e^4) - (-e^{-2} - e^2 + 2)$$

$$= \cancel{e^2} + e^6 - e^4 - e^4 - e^{-2} - \cancel{e^2} + 2$$

$$= e^6 - e^4 - e^4 - e^{-2} + 2$$

13. $I = \int_C \sin(x+y)\,dx + (3x+y)\,dy$



$$\int_0^2 \int_0^6 3 - \cos(x+y) \cdot 1 \, dy\,dx$$

$$3y + \sin(x+y) \Big|_0^6 = (18 - \sin(6+x)) - (0 - \sin(x))$$

$$\int_0^2 18 - \sin(6+x) + \sin(x)\,dx = 18x + \cos(6+x) - \cos(x) \Big|_0^2$$

$$= (18(2) + \cos(8) - \cos(2)) - (0 + \cos(6) - \cos(0))$$

$$= 36 - 2$$

$$= 34$$

17.2

1. $F = \langle 2xy, x, y+z \rangle$ $\quad z = 1 - x^2 - y^2$ for $x^2+y^2 \le 1$

$2 \le 1 - x^2 - y^2 \quad z + x^2 + y^2 = 1 \quad z - 1 \le 1$

$\iint_C F \cdot ds = \iint_S curl(F) \cdot \vec{n}\, dS \quad \hookrightarrow x^2+y^2 \le 1 \quad z \le 2$

$x = \cos t \quad y = \sin t \quad z = 2$

$F = \langle 2\cos t \sin t, \cos t, \sin t + \cos t \rangle \quad -z = -1 + x^2 + y^2$

$z = -1 + 1 = 0$

$r(t) = \langle \cos t, \sin t, 2 \rangle$

$dr(t) = \langle -\sin t, \cos t, 0 \rangle$

$F \cdot dr = -2\cos t \sin^2 t + \cos^2 t\, dt$

$\int_0^{2\pi} -2\cos t \sin^2 t + \cos^2 t\, dt = -\frac{2}{3}\sin^3 t + \frac{1}{3}\cos^3 t \Big|_0^{2\pi}$

$= \left(\frac{-2}{3}\sin^3(2\pi) + \frac{1}{3}\cos^3(2\pi)\right) - \left(\frac{-2}{3}\sin^3 0 + \frac{1}{3}\cos^3(0)\right)$

$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

3. $F = \langle e^{y-z}, 0, 0 \rangle \quad 0 \le x \le 1, \quad 0 \le y \le 1 \quad 0 \le z \le 1$

$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{y-z} & 0 & 0 \end{vmatrix} = (0 - 0)i - (0 + e^{y-z})j + (0 - e^{y-z})k$

$= -e^{y-z} j - e^{y-z} k$

$\int_0^1 \int_0^1 e^{y-z} - e^{y-z}\, dy\, dz$

$= e^{y-z} - e^{y-z} \Big|_0^1 = \int_0^1 e^{1-z} - e^{1-z}\, dz = -e^{1-z} + e^{1-z} \Big|_0^1$

$= e^{-1} - 1$

5. $F = \langle e^{z^2} - y, e^{z^3} + x, \cos(xz) \rangle$  $x^2 + y^2 + z^2 = 1$  $z \geq 0$

$z = \sqrt{1 - x^2 - y^2}$

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ e^{z^2} - y & e^{z^3} + x & \cos(xz) \end{vmatrix} = (0 - 3z^2 e^{z^3})i - (\cos(xz) - 2z \cdot e^{z^2})j + (1+1)k$$

$$= -3z^2 e^{z^3} i - (\cos(xz) - 2ze^{z^2})j + 2k$$

$$\int (1 + \sin t, 1 + \cos t, \cos(0)) \cdot \langle -\sin t, \cos t, 0 \rangle \, d\theta$$

$$\int_0^{2\pi} 0 + (-\sin t - (\sin + \cos t)) \, d\theta$$

$$\int_0^{2\pi} 0 \, d\theta = 2\pi - 0 = 2\pi$$

9. $F = \langle yz, xz, xy \rangle$  $x^2 + y^2 = 1$  $z = 1$ and $z = 4$

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ yz & xz & xy \end{vmatrix} = (x - x)i - (y - y)j + (z - z)k$$

$$= 0i + 0j + 0k = 0$$

11. $F = \langle 3y, -2x, 3y \rangle$  $x^2 + y^2 = 9$  $z = 2$

$F = \langle 9\sin t, -6\cos t, 9\sin t \rangle$  $x = 3\cos t$  $y = 3\sin t$  $z = 2$

$r(t) = \langle 3\cos t, 3\sin t, 2 \rangle$

$r'(t) = \langle -3\sin t, -3\cos t, 0 \rangle$

$$\int_0^{2\pi} F \cdot dr = \int 27\sin^2 t + 18\cos^2 t + 0$$

$$9\sin^3 t + 6\cos^3 t \Big|_0^{2\pi} = 9\sin^3 2\pi + 6\cos^3 2\pi - (0 + 6\cos^3(0))$$

$$= 6 - 6 = 0$$

13. $F = \langle y, z, x \rangle$ $(0,0,0)$ $(3,0,0)$ $(0,3,3)$

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ y & z & x \end{vmatrix} = (0-1)i - (1-0)j + (0-1)k$$

$$= -i - j - k \quad \langle -1, -1, -1 \rangle$$

$$= 0$$