17.1.

1. $\oint_C xy\,dx + y\,dy$

$P = xy,\ Q = y.$

$\cancel{\iint x\,dx\,dy}$

$\oint_C xy\,dx + y\,dy$

$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \left(\frac{\partial}{\partial x} y - \frac{\partial}{\partial y} xy\right) dy\,dx.$

3. $P = y^2 \quad Q = x^2$

$\oint_C y^2\,dx + x^2\,dy$

$= \int_0^1\int_0^1 (2x - 2y)\,dx\,dy$

$= \int_0^1 (1 - 2y)\,dy$

$= 1 - 1$

$= 0.$

5. $\iint -x^2\,dA$

$= \int_0^1\int_0^{2\pi} -r^3\cos^2\theta\,d\theta\,dr.$

$= \pi\int_0^1 -r^3\,dr$

$= -\frac{\pi}{4}.$

7. $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2x.$

$\int_0^1\int_{x^2}^{x} 2x\,dy\,dx$

$= \int_0^1 (2x^2 - 2x^3)\,dx$

$= \left[\frac{2}{3}x^3 - \frac{1}{2}x^4\right]_0^1$

$= \frac{1}{6}$

9. $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = e^{x-y} - e^{x+y}$

$-\left(\int_0^2\int_0^x (e^{x-y} - e^{x+y})\,dy\,dx\right.$

$\left.+ \int_2^4\int_{x-2}^{2} (e^{x-y} - e^{x+y})\,dy\,dx\right)$

$= \frac{e^6}{2} - \frac{e^4}{2} - \frac{5e^2}{2} + \frac{5}{2}.$

13. $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2$

For ABCDA part.

$2\cdot\frac{[(4-2)+6]\times 2}{2} = 16.$

DA part.

$\int_6^0 t\,dt = \left[\frac{t^2}{2}\right]_6^0 = -18$

$I = 16 - (-18) = 34.$

17.2

1. $\cancel{\nabla\times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & x & y+z \end{vmatrix}}$

$= \cancel{i + (1-2x)k}$

$x^2+y^2=1$

$r=\langle \cos t, \sin t, 0\rangle$

$dr=\langle -\sin t, \cos t, 0\rangle$

$F=\langle 2\sin t\cos t, \cos t, \sin t\rangle$.

$\int_0^{2\pi} F\cdot dr = \int_0^{2\pi} (-2\sin^2 t\cos t + \cos^2 t)\,dt$

$= \pi$

3. $\text{curl}F = 0i + e^{y-z} j - e^{y-z} k$.

when $z=1$.

$\iint_S \text{curl}F\cdot dS = \int_0^1\int_0^1 -e^{y-1}\,dx\,dy$

$= e^{-1}-1$.

5. $\text{curl}F = i(-3z^2e^{z^3}) - j(-z\sin xz - 2ze^{z^2}) + 2k$

$= -3z^2e^{z^3} i + (z\sin xz + 2ze^{z^2}) j + 2k$

$r=\langle \cos t, \sin t, 0\rangle$

$dr=\langle -\sin t, \cos t, 0\rangle$

$F=\langle 1-\sin t, 1+\cos t, 1\rangle$

$\int_0^{2\pi} F\cdot dr = \int_0^{2\pi} (\sin^2 t + \cos^2 t + \cos t - \sin t)\,dt$

$= \cancel{2\pi}\, 2\pi + 1 - 1 = 2\pi$

7. $\text{curl}F = -2i + 3j + 5k$

$\cancel{r=\langle 2\cos t, 2\sin t, 4\rangle}$

$\cancel{dr=\langle -2\sin t, 2\cos t, 0\rangle}$.

$\cancel{F=\langle 12, 10\cos t, -4\sin t\rangle}$.

$\cancel{\int_0^{2\pi} F\cdot dr = \int_0^{2\pi} (-24\sin t + 20\cos t)\,dt}$.

when $z=4$

$\iint_S \text{curl}F\cdot dS = \int_0^2\int_0^2 5\,dx\,dy$

$= 5\times 2\times 2$

$= 20$.

11. $r=\langle 3\cos t, 3\sin t, 2\rangle$.

$dr=\langle -3\sin t, 3\cos t, 0\rangle$.

$F=\langle 9\cos t, -6\sin t, 9\cos t\rangle$.

$\int_0^{2\pi} F\cdot dr = \int_0^{2\pi} -27\sin^2 t - 18\cos^2 t\,dt$

$= -45\pi$

13. $\text{curl}F = -i - j + k$.

$g(x,y) = y$

$\iint_C F\cdot dr = \int_0\int_0^3 (1+1)\,dx\,dy$

$\cancel{= 2\times 3\times 2}$

$\cancel{= 12}$

$= 0$