

HW 17.1-17.2

duc 12/6/20

17.1

$$1. \oint xy dx + y dy \rightarrow \int_0^{2\pi} \cos\theta \sin\theta (-\sin\theta) + \sin\theta \cos\theta d\theta \rightarrow 0$$

$$+ \int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (0-x) dy dx = \iint_D \left(\frac{\partial}{\partial x} y - \frac{\partial}{\partial y} xy \right) dA$$

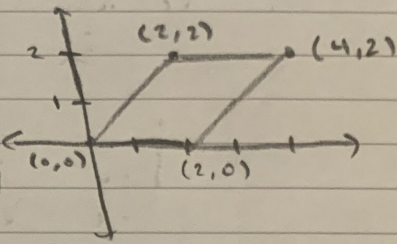
$$3. \oint y^2 dx + x^2 dy \quad P = y^2 \quad Q = x^2 \quad \frac{\partial}{\partial y} y^2 = 2y \quad \frac{\partial}{\partial x} x^2 = 2x$$

$$\iint 2x - 2y dA \rightarrow \int_0^1 \int_0^1 2x - 2y dx dy \rightarrow \boxed{0}$$

5. $\oint x^2 y dx$ $P = x^2 y$ $\frac{\partial}{\partial y} x^2 y = x^2$
 $\int_0^{2\pi} \int_0^1 r^3 \cos^2 \theta dr d\theta \rightarrow \int_0^{2\pi} \frac{r^4}{4} \cos^2 \theta |_0^1 d\theta$
 $\int_0^{2\pi} \frac{1}{4} \cos^2 \theta d\theta \rightarrow \boxed{-\pi/4}$

7. $\oint F \cdot dr$, where $F(x^2, x^2)$ $C = y = x^2$ $y = x$ $0 \leq x \leq 1$
 $\int_0^1 \int_{x^2}^x 2x dy dx \rightarrow \int_0^1 2xy |_{x^2}^x dx = \int_0^1 2x^2 - 2x^3 dx$
 $\frac{2x^3}{3} - \frac{x^4}{2} |_0^1 = \boxed{\frac{1}{6}}$

9. $F(x, y) = \langle e^{x+y}, e^{x-y} \rangle$ $y \leq x \leq y+2$
 $\iint (e^{x-y} - e^{x+y}) dA$ $0 \leq y \leq 2$
 $\int_0^2 (e^{-y}(e^{2+y} - e^y) - e^y(e^{2+y} - e^y)) dy$
 $= (2e^2 + \frac{e^4 + e^2 - e^6 - 1}{2} - 2) - 1 \rightarrow$



$\boxed{2 - \frac{e^4 + e^2 - e^6 - 1}{2} - 2e^2}$

13. $\int_C (\sin(x+y) dx + (3x+y) dy)$ $P = \sin(x+y)$ $Q = 3x+y$
 $\frac{\partial}{\partial x} (3x+y) - \frac{\partial}{\partial y} (\sin(x+y)) = 3 - 1 = 2$
 $\iint 2 dA$ \star Don't know where to go from here
 $\int_0^6 ? dy$

17.2

1. $F = \langle 2xy, x, y+z \rangle$ $z = 1 - x^2 - y^2$ for $x^2 + y^2 \leq 1$

$r(t) = r \cos t i + r \sin t j + z k$

$r'(t) = -r \sin t i + r \cos t j + 0 k$

$\iint \text{curl } F \cdot ds = \int F \cdot dr = \int_0^{2\pi} F(r(t)) \cdot r'(t) dt = \boxed{\pi}$

$F = \langle e^{y-z}, 0, 0 \rangle$ square $(1,0,1)$, $(1,1,1)$, $(0,1,1)$, $(0,0,1)$

$\text{curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{y-z} & 0 & 0 \end{vmatrix}$ $e^{y-z} + e^{y-z} \quad 2e^{y-z}$

$\iint 2e^{y-z} \frac{\nabla F}{\|\nabla F\|} \cdot \mathbf{n} dA = \boxed{e^{-1} - 1}$

$$5. F = \langle e^{z^2} - y, e^{z^2} + x, \cos(xz) \rangle \quad x^2 + y^2 + z^2 = 1, z \geq 0$$

$$\text{curl } F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{z^2} - y & e^{z^2} + x & \cos(xz) \end{vmatrix} = \langle -3ze^{z^2}, 2ze^{z^2} + z \sin(xz), z \rangle$$

$$\int \langle -3ze^{z^2}, 2ze^{z^2} + z \sin(xz), z \rangle \cdot \nabla f \, dA = \boxed{2\pi}$$

$$9. F = \langle yz, xz, xy \rangle \quad x^2 + y^2 = 1 \quad z = 1, 4$$

$$\int \text{curl } F \cdot \nabla f \, dA$$

$$\text{curl } F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} \rightarrow \boxed{0}$$

$$11. F = \langle 3y, -2x, 3y \rangle \quad x^2 + y^2 = 9, z = 2$$

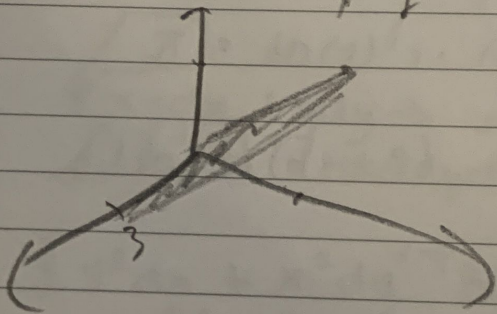
$$\text{curl } F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3y & -2x & 3y \end{vmatrix} = \langle 3, 0, -5 \rangle$$

$$\int \text{curl } F \cdot \nabla f \, dA$$

$$\int \langle 3, 0, -5 \rangle \cdot \nabla f \, dA = \boxed{45\pi}$$

$$13. F = \langle y, z, x \rangle \quad (0, 0, 0), (3, 0, 0), (0, 3, 3)$$

$$\text{curl } F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z & x \end{vmatrix} = \langle -1, -1, -1 \rangle$$



$$\int \text{curl } F \cdot \nabla f \, dA$$

$$\int \langle -1, -1, -1 \rangle \cdot \nabla f \, dA = \boxed{0}$$