17.1 Homework

1. $\oint_c xy\,dx + y\,dy$

$x = \cos t \quad y = \sin t \quad 0 \le t \le 2\pi$

$dx = -\sin t\,dt \quad dy = \cos t\,dt$

$\int_0^{2\pi} (-\sin^2 t \cos t + \sin t \cos t)\,dt$ Using maple: $\boxed{0}$

Green's Theorem:

$P = xy \quad Q = y$

$\frac{\partial P}{\partial y} = x \quad \frac{\partial Q}{\partial x} = 0 \quad \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0 - x = -x$

Counter-clockwise = positive

Unit-circle: $x^2 + y^2 \le 1$

$\iint -x\,dA = \int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} -x\,dx\,dy = 0$ ✓

3. $\oint_c y^2\,dx + x^2\,dy \quad 0 \le x \le 1 \quad 0 \le y \le 1$

$P = y^2 \quad Q = x^2$

$\frac{\partial P}{\partial y} = 2y \quad \frac{\partial Q}{\partial x} = 2x \quad \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2x - 2y$

$\iint_D 2x - 2y\,dA = \int_0^1\int_0^1 (2x-2y)\,dx\,dy = \boxed{0}$

5. $\oint_C x^2y\,dx$ C is unit circle

$P = x^2y$ $Q = 0$

$\frac{\partial P}{\partial y} = x^2$ $\frac{\partial Q}{\partial x} = 0$

$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = -x^2$

$x^2+y^2=1$ $r^2=1$ $0 \le r \le 1$

$0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_0^1 (x^2)\,r\,dr\,d\theta = \int_0^{2\pi}\int_0^1 (r^2\cos^2\theta)\,r\,dr\,d\theta = \frac{-\pi}{4}$$

7. $\oint_C F \cdot dr$ $F(x,y) = \langle x^2, x^2\rangle$ C consists of arcs $y=x^2$ and $y=x$ $0 \le x \le 1$

$P = x^2$ $Q = x^2$

$\frac{\partial P}{\partial y} = 0$ $\frac{\partial Q}{\partial x} = 2x$ $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2x$

$$\iint_D 2x\,dA = \int_0^1\int_x^{x^2} 2x\,dy\,dx = \boxed{\frac{1}{6}}$$

9. $P = e^{x+y}$ $Q = e^{x-y}$

$\frac{\partial P}{\partial y} = e^{x+y}$ $\frac{\partial Q}{\partial x} = e^{x-y}$ Clockwise.

$$\iint_D e^{x-y} - e^{x+y}\,dA$$

I think here you split up into multiple shapes.
But, I'm not entirely sure.

11. $F\langle x,y\rangle = \langle 2xe^y,\ x+x^2e^y\rangle$ C is quarter circle

P Q

$\frac{\partial P}{\partial y} = 2xe^y \qquad \frac{\partial Q}{\partial x} = 1+2xe^y$

$(1+2xe^y) - 2xe^y = 1$

$\iint_D 1\, dA$

Since it is a quarter circle, $0 \le r \le 4 \qquad 0 \le \theta \le \pi/2$

$\int_0^{\pi/2}\int_0^4 r\,dr\,d\theta \quad \frac{r^2}{2}\Big|_0^4 = 8$

$8 \cdot \int_0^{\pi/2} d\theta = 8\theta \Big|_0^{\pi/2} = \frac{8\pi}{2} = 4\pi$

17.2 Homework

1. $F = \langle 2xy, x, y+z \rangle$ $\quad z = 1 - x^2 - y^2$

$x^2 + y^2 \le 1$

$r(t) = \langle \cos t, \sin t, 0 \rangle \quad 0 \le t \le 2\pi$

$r'(t) = \langle -\sin t, \cos t, 0 \rangle$

$F(x,y,z) = \langle 2\sin t \cos t, \cos t, \sin t \rangle$

$$\oint F \cdot dr = \int_0^{2\pi} (-2\sin^2 t \cos t + \cos^2 t)\, dt$$

$\Downarrow$

This is just $\boxed{\pi}$

3. $F = \langle e^{y-z}, 0, 0 \rangle$ square with vertices A $(1,0,1)$, B $(1,1,1)$, C $(0,1,1)$, D $(0,0,1)$

$$\text{curl}(F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{y-z} & 0 & 0 \end{vmatrix} = \hat{i}[(0-0)] - \hat{j}[0 + e^{y-z}] + \hat{k}[-e^{y-z}]$$

$\text{curl}(F) = -e^{y-z}\hat{j} - e^{y-z}\hat{k}$

$\text{curl}(F) \cdot ds = -e^{y-z}$

$z = 1$, so $\quad -e^{y-1}$

$$\int_0^1 -e^{y-1}\, dy = -e^{y-1}\Big|_0^1 = -1 + e^{-1}$$

5. $F = \langle e^{z^2} - y, e^{z^3} + x, \cos(xz) \rangle$

$$\text{curl}(F) = \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{matrix}$$

$\hat{i}(0 - 3z^2 e^{z^3}) - \hat{j}(-z\sin(xz) - 2ze^{z^2}) + \hat{k}(2)$

$\langle -3z^2 e^{z^3}, -z\sin(xz) - 2ze^{z^2}, 2 \rangle$

Kind of stuck here.

9. $F = \langle yz, xz, xy \rangle$

$$\text{curl}(F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & xz & xy \end{vmatrix}$$

$$\hat{i}(x-x) - \hat{j}(y-y) + \hat{k}(z-z) = \langle 0,0,0 \rangle$$

Therefore, the flux is 0.

11. $F = \langle 3y, -2x, 3y \rangle$ C is the circle $x^2+y^2=9$, $z=2$ counterclockwise

$$\text{curl}(F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3y & -2x & 3y \end{vmatrix}$$

$$\hat{i}(3-0) - \hat{j}(0-0) + \hat{k}(-2-3)$$

$$\langle 3, 0, -5 \rangle$$

$$\int_S \text{curl} F \cdot dS = \int_S (3\hat{i} - 5\hat{k}) \cdot dS$$

I went wrong somewhere, and I am not sure what to do next.

13. $F = \langle y, z, x \rangle$ C is triangle with $(0,0,0)$ $(3,0,0)$ $(0,3,3)$

$$\text{curl}(F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & z & x \end{vmatrix}$$

$$\hat{i}(0-1) - \hat{j}(1-0) + \hat{k}(1-0)$$

$$-\hat{i} - \hat{j} + \hat{k}$$

$$\langle -1, -1, 1 \rangle$$

$P = (0,0,0) \quad Q = (3,0,0) \quad R = (0,3,3)$

$PQ = \langle 3,0,0 \rangle \qquad PR = \langle 0,3,3 \rangle$

$$PQ \times PR = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ 0 & 3 & 3 \end{vmatrix}$$

$\hat{i}[0-0] - \hat{j}[9-0] + \hat{k}[9-0]$

$\langle 0, -9, 9 \rangle = N$

Point: $(0,0,0)$

$\cancel{0(x-0)} + 9(y-0) + 9(z-0) = 0$

$-9y + 9z = 0$

$9z = 9y \qquad z = y$

$\text{curl}(F) = \langle -1, 1, 1 \rangle$

$\int_S -\hat{i} + \hat{j} + \hat{k} \cdot dS \qquad z = y$

$y = y$

$\int_D 0 + \hat{j} + \hat{k} \cdot dA$

$\iint 2 \cdot dA$

$2 \cdot \int_0^3 \int_0^{3-x} dA$

$2 : \int_0^3 \int_0^{3-x} dy\,dx = y \Big|_0^{3-x} = \int 3-x\,dy = 3x - \frac{x^2}{2} \Big|_0^3$

$\boxed{9 - \frac{9}{2}}$