17.1 & 17.2 Homework

17.1

1. $\iint_D -x \, dA = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} -x \, dy\,dx = 0$

$\int_0^{2\pi}\int_0^1 -r^2\cos\theta \, dr\,d\theta = 0$

3. $= \iint_D 0 \, dA = \boxed{0}$

5. $\iint_D -x^2 \, dA = \int_0^{2\pi}\int_0^1 -r^3\cos^2\theta \, dr\,d\theta$

$= \left(-\int_0^1 r^3 dr\right)\left(\int_0^{2\pi} \cos^2\theta \, d\theta\right) = \left(-\frac{1}{4}\right)\left(\int_0^{2\pi} \frac{\cos 2\theta + 1}{2}\right)$

$= \left(-\frac{1}{4}\right)\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\Big|_0^{2\pi}$

$= \left(-\frac{1}{4}\right)(\pi) = \boxed{-\frac{\pi}{4}}$

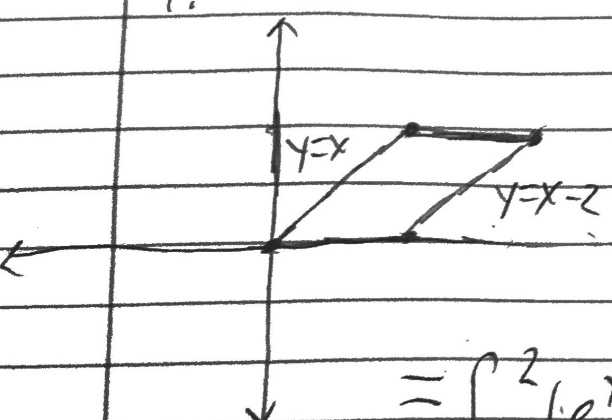
7. $\oint_C x^2dx + x^2dy$

$= \int_0^1\int_{x^2}^{x} 2x\,dy\,dx = \int_0^1 2xy\Big|_{x^2}^{x} dx$

$= \int_0^1 2x^2 - 2x^3\,dx = \frac{2}{3}x^3 - \frac{x^4}{2}\Big|_0^1$

$= \frac{2}{3} - \frac{1}{2} = \boxed{\frac{1}{6}}$

9.



$\int_0^2\int_y^{y+2} e^{x-y} - e^{x+y}\,dx\,dy$

$= \int_0^2 e^{x-y} - e^{x+y}\Big|_y^{y+2} dy$

$= \int_0^2 (e^x)(e^{-y} - e^y)\Big|_{x=y}^{x=y+2} dy$

$= \int_0^2 (e^{y+2} - e^y)(e^{-y} - e^y)\,dy$

$\int_0^2 e^2 - e^{2y+2} - e^2 - e^{2y}\,dy$

$= \int_0^2 (e^2 - 1)(1 - e^{2y})\,dy$

$(e^2-1)\left(y - \frac{e^{2y}}{2}\right)\Big|_0^2$

$(e^2-1)\left(2 - \frac{e^4}{2} + \frac{1}{2}\right) = -158.443 \rightarrow \boxed{158.443}$

~~(clockwise)~~

13. $= \iint_D 3-1\,dA = \iint_D 2\,dA$

$= (2)\left(\frac{1}{2}\right)(2+6)(2) = 16$

$\langle 0, t\rangle \quad 0 \le t \le 6$

$\int_0^6 t\,dt = \frac{t^2}{2}\Big|_0^6 = 18 \rightarrow -18$ (clockwise)

$16-(-18) = \boxed{34}$

17.2

Stokes' Theorem

1. $r(t) = \cos\theta\, i + \sin\theta\, j + 0k$

$F(r(t)) = \langle 2\sin\theta\cos\theta, \cos\theta, \sin\theta\rangle$

$r'(t) = \langle -\sin\theta, \cos\theta, 0\rangle$

$\int_0^{2\pi} F(r(t)) \cdot r'(t) = \int_0^{2\pi} -2\sin^2\theta\cos\theta + \cos^2\theta$

Used maple, $\rightarrow \boxed{\pi}$

Normal way

They are the same

$\text{curl}(F) = i(z-0) - j(0-0) + k(1-2y)$

$\langle z, 0, 1-2y\rangle = \langle 1-x^2-y^2, 0, 1-2y\rangle$

$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (1-x^2-y^2)(-2x) + 1-2y \; dy\,dx$

Used maple $\rightarrow \boxed{\pi}$

3. Stokes' Theorem

$\iint \text{curl}(F) \cdot dS = \int F \cdot dr$

$z=1$ $\quad r(t) = \langle t, t, 1 \rangle$

$x=t$ $\quad r'(t) = \langle 1, 1, 1 \rangle$

$y=t$

$= \int \langle e^{t-1}, 0, 0 \rangle \cdot \langle 1, 1, 1 \rangle \, dt$

$= \int_0^1 e^{t-1} \, dt \quad \boxed{= 1 - e^{-1}}$

Normal Way

$\text{curl}(F) = \langle 0, -e^{y-z}, -e^{y-z} \rangle$

$\int_0^1 \int_0^1 -e^{y-1} \, dy\,dx \quad \boxed{= e^{-1} - 1}$

5. $\text{curl}(F) = \langle -3z^2 e^{z^3}, x\sin(xz) + 2ze^{z^2}, 2 \rangle$

$r(\theta) = \cos\theta i + \sin\theta j + 0k \quad 0 \le t \le 2\pi$

$r'(\theta) = \langle -\sin\theta, \cos\theta, 0 \rangle$

$F(r(\theta)) = \langle -\sin\theta, \cos\theta, 1 \rangle$

$\int_0^{2\pi} F(r(\theta)) \cdot r'(\theta) = \int_0^{2\pi} \sin^2\theta + \cos^2\theta \, d\theta$

$\boxed{= 2\pi}$

9. $\text{curl}(F) = i(x-x) - j(y-y) + k(z-z)$
$= \langle 0,0,0 \rangle$, conservative vector field

$\iint \text{curl}(F) \cdot dS = \boxed{0 \text{ (zero vector)}} = \int F \cdot dr$

11. $r(\theta) = \langle 3\cos\theta, 3\sin\theta, 2 \rangle \quad 0 \le \theta \le 2\pi$
$r'(\theta) = \langle -3\sin\theta, 3\cos\theta, 0 \rangle$

$F(r(\theta)) = \langle 9\sin\theta, -6\cos\theta, 9\sin\theta \rangle$

$\int_0^{2\pi} F(r(\theta)) \cdot r' = \int_0^{2\pi} -27\sin^2\theta - 18\cos^2\theta$

used maple $\rightarrow \boxed{-45\pi}$

13. Plane: $-9y + 9z = 0$ $\quad \langle 36, 36, 36 \rangle$
$y = z$

$\iint \text{curl}(F) = \int F \cdot dr$ $\quad \boxed{= 0 \text{ It's a closed surface!}}$