$$\oint P\,dx + Q\,dy = \iint_R \frac{dQ}{dx} - \frac{dP}{dy}$$

$\iint$ integrals are easy to deal with if non-conservative fields.

Only on non-Conservative vector fields $\left(\frac{dQ}{dx} = \frac{dP}{dy}\right)$

$P\hat{i} + Q\hat{j}$

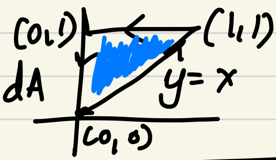
$r(t) = x\hat{i} + y\hat{j}$

$dr = dx\,\hat{i} + dy\,\hat{j}$

$W = F \cdot dr = (P\hat{i} + Q\hat{j}) \cdot (dx\,\hat{i} + dy\,\hat{j})$

$= \int P\,dx + Q\,dy$

(1) $\oint x^3 dx + \cancel{x}y\,dy$ $\quad (0,0)\ (1,1)\ (0,1)$



$$\oint x^3 dx + xy\,dy = \iint_R (y-0)\,dA$$

$$\boxed{\int_0^1 \int_0^y y\,dx\,dy}$$

$\boxed{x = 0 \text{ to } x = y \quad y = 0 \text{ to } 1}$

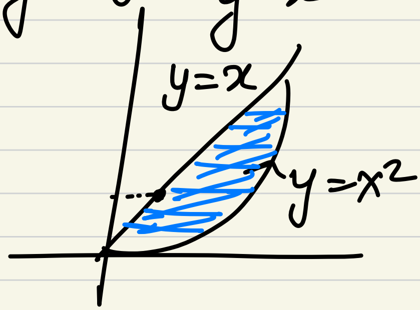
$= \frac{1}{3}$

$$\boxed{\left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dA}$$

② $\int x^2 y + x^3 \, dx + 2xy \, dy$ $\quad y = x$ and $y = x^2$

$$\iint (2y - x^2) \, dA$$

$$\iint (2y - x^2)$$



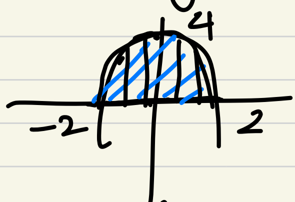
$$\boxed{\int_0^1 \int_{x^2}^{x} (2y - x^2) \, dy \, dx}$$

$y = x$ to $y = x^2$

$x = 0$ to 1

③ $\oint (\underbrace{y^2 + \cos x}_{P}) dx + (\underbrace{x - \tan^{-1} y}_{Q}) dy$ $\quad y = 4 - x^2$, $y = 0$

$$\iint_R (1 - 2y) \, dA$$

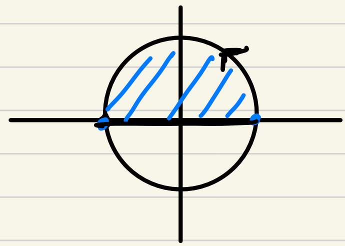


$$\boxed{\int_{-2}^{2} \int_0^{4-x^2} (1 - 2y) \, dy \, dx}$$

$x = -2$ to 2

$y = 0$ to $4 - x^2$

4. $\oint x^2y\,dx + y^3\,dy$ C: $(-1,0) \to (1,0)$, $x^2+y^2=1$



Homework:

9. $$F(x,y) = \langle \overset{P}{e^{x+y}}, \overset{Q}{e^{x-y}} \rangle$$

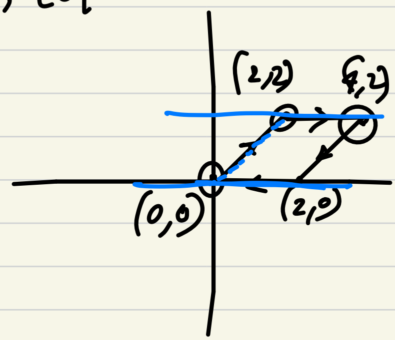
Please Check

$(0,0)$, $(2,2)$, $(4,2)$, $(2,0)$, $(0,0)$

$$\oint \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dA$$

$$\oint e^{x-y} - e^{x+y}\, dA$$

$$\int_0^4 \int_y^{y+2} e^{x-y} - e^{x+y}\, dx\, dy$$



$$\boxed{\text{My ans} = \left(\frac{-9e^2+9+e^{10}-e^8}{2}\right)}$$

What went wrong?

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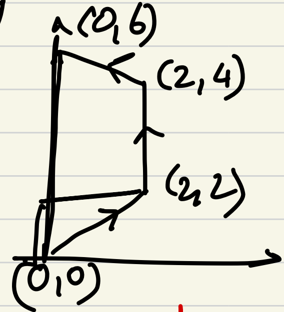
$$\oint \overset{P}{(\sin x + y)}\,dx + \overset{Q}{(3x + y)}\,dy$$



$$\iint_R (1 - 3)\, dA$$

$$\iint_R (-2)\, dA$$

→ What is the integrand of the line integral?

$$\int_0^6$$